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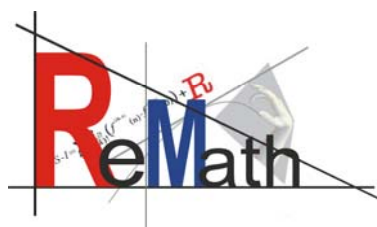
Programme

Project Number: IST4-26751

ReMath

Representing Mathematics with Digital Media

DESIGN-BASED RESEARCH: PROCESS AND RESULTS

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Introduction

Within the ReMath Project, WP4 is in charge of developing the design and coordinating the implementation of Empirical Research which is expected to contribute to the Project in different ways:

1. providing "the validation of the D.D.A.'s both in respect to their functioning as didactical tools and the consistency of such functioning in relation to the theoretical assumptions, on which scenarios (tools and activities) have been designed" (Project Proposal part B, p. 59);

2. producing common results that enhance "our understanding of meaning-making through representing with digital media" (ibidem, p. 9), in particular offering the project new perspectives at very general level, by providing new insight on a) means of using technologies to support learning b) design issues concerning both DDA and scenarios, and c) learning processes in relation to the use of technologies;

3. "providing validation for the [integrative theoretical] framework" (ibidem, p. 8). In fact, the Integrative Theoretical Framework (ITF) produced in WP1 is expected to provide a methodological tool for attaining the aims above and, conversely, the experimentation carried out within WP4 is meant to validate the effectiveness of the ITF and possibly to provide elements for refining the ITF itself.

In order to carry out the design and coordination of the empirical research for attaining the stated objectives, WP4 is in charge of organizing and coordinating the following key parts of the experimentation:

- the design of Teaching Experiments based on the six different DDAs that are developed in WP2 and on the different Pedagogical Plans constructed in WP3;

- the implementation of the Teaching Experiments in "real" learning situations (which could be regular classes but also informal settings of learning, such as teachers training);

- the collection of data and their analysis;

- the comparison and possible integration of the findings coming from the different Teaching Experiments.

Obviously, the planning and coordination of the teaching experiments were organized according to the objectives of WP4 within the Project.

A prior need was identified in creating the conditions for addressing the issues raised by the stated objectives in an effective way. In particular the aim of producing **common results** "enhancing our understanding of meaning-making through representing with digital media" raised the issue of comparing and possibly integrating the results of the different teaching experiments.

The WP4 previous deliverable, Del11, accounts the whole process of setting up a methodology for efficiently address those issues. In particular, we endeavoured to design a methodological tool for supporting the experimenting teams in their effort of precisely defining each experimental design and research questions in relation to the Project objectives, so that the implementation of the single teaching experiment can produce a set of comparable and significant results. This process led us to the construction of the Guidelines, which, once filled, originated the so-called Teaching Experiment Portraits. In the Del 11 we developed a first analysis of the Teaching Experiment Portraits intending to:

1. Review each experimental design, check their internal consistency, make possible implicit elements and discrepancies emerge so to start a process of refinement of the Portraits themselves. This process aimed at producing experimental designs which could ensure the possibility of effectively pursuing all the objectives of WP4, and in particular at creating the conditions under which the implementation of the Teaching Experiments could produce a set of comparable and significant results.

2. Define possible paths for future analysis to be carried out in order to identify possible similarities and complementarities among the ways in which the different Teaching Experiments address the common issues of the Project.

3. Try to envisage the possible contributions coming from each Teaching Experiment to the general issues raised and to "place" those contributions with respect to each other, specifying the aspects to which the different results could be related.

This deliverable is meant to report on the results of the teaching experiments, as well as to develop a comparison among those results. The first chapter recalls the overall organization of the experimentation, including the description of the Teaching Experiment Guidelines, and the rationale according to which they were set up, which links them to WP4 objectives. The second chapter describes the methodological tools jointly set up in order to organize the report on the results coming from each Teaching Experiment according to a common frame, the so-called Teaching Experiment Analysis Guidelines. Details about the actual implementation of the designed Pedagogical Plans and related issues are discussed in the third chapter. The fourth chapter develops the analysis of TEs results with respect to the aim of providing feedbacks on the DDAs-centred Pedagogical Plans implemented. The fifth chapter focuses on the answers provided to the Reformulated Common Research Questions addressed through the different Teaching Experiments. In this chapter a comparison among those answers is developed as well, with the aim of producing common results enhancing our understanding of meaning-making through representing with digital media. The sixth chapter discusses the perspectives for further possible investigations which can especially feed the development of the ITF within the WP1.

I. Experimental design: overview

The experimentation carried out within WP4 involved different teaching experiments, each based on a Pedagogical Plan developed in WP3 and centred on the use of at least one of the DDAs developed in WP2.

The different teaching experiments were coordinated through a *Cross Experimentation*. The Cross Experimentation is a methodological approach designed with the aims of gaining a deeper understanding across research teams with diverse practices and cultures, and of progressing towards integrated views of technology use in math education.

As clearly expressed in ReMath Project Proposal, all the partners involved in the research activity agreed to conduct teaching experiments centred on the use of the DDAs that they produced ("*familiar DDAs*"), if any, and teaching experiments centred on the use of the DDAs that they did not produce ("*alien DDA*"). One more self-imposed constraint is that each DDA will be experimenting both by a team involved in the DDA design and by a team not involved in the process. This shared decision led to the planning of 13 teaching experiments. The following table indicates for each DDA the team which experiments it as familiar DDA (these teams participated in the DDA design process) and the teams which experiment it as alien DDA.

DDA	Team experimenting the DDA as *familiar* DDA	Team experimenting the DDA as *alien* DDA
Aplusix	MeTAH	ITD
		UNISI
Alnuset	ITD	MeTAH
Casyopée	DIDIREM	UNISI
Cruislet	ETL	DIDIREM
MaLT	ETL	IOE
MoPiX	IOE	ETL

Table 1. Organization of the Cross-Experimentation

As it appears from the table, the couplings DDAs-Teams is asymmetric. This happens because on the one hand, ETL participated to the design process of two different DDAs, namely Cruislet and MaLT, and hence it will conduct three teaching experiments (two based on the use of these familiar DDAs and one based on the use of an alien DDA). On the other hand, UNISI team did not participate to the design of any DDA, and so it will conduct two teaching experiments both based on alien DDAs.

I.1. Teaching Experiments

With the aim of addressing our objectives within the complexity of the learning process, we chose to employ the research method of Teaching Experiments (Steffe and Thompson, 2000).

According to this reserach method,teaching sequences are taken into consideration together with their whole *learning ecology* (Cobb et al. 2003, p.10), i.e. that complex and interacting system of factors influencing learning such as classroom activities, tasks, problems, social norms, tools used, teachers' actions and so on.

Teaching experiments are *experiments* in the sense that they are meant to validate hypotheses, refine Hypotheses, re-formulate hypotheses… We are by no means relating the use of the term "experiment" to "control/experimental group" research methods widely spread in social, behavioural and clinical studies.

One distinctive common feature of teaching experiments is their *iterative nature*: theoretical hypotheses drive the design of a teaching intervention, whose implementation and evaluation allow to validate or refine (possibly reject) these hypotheses.

In a Teaching Expriment, the continuous generation of hypotheses appears in a sense inevitable. In fact, as Collins et al. pointed out,

> "any implementation of a design requires many decisions that go beyond the design itself. This occurs because no design can specify all the details, and because the actions of participants in the implementation […] require constant decisions about how to proceed at every level." (Collins et al. 2004, p.17)

Of course, in order to carry out the described iterative process it is necessary to make explicit at least the hypotheses motivating such contingent decisions.

I.1.1 Teaching Experiment methodology and ReMath objectives

The main features of the Teaching Experiment methodology suggest that it may fit well the objectives of WP4, as envisaged in the Project Proposal. Let us see more in details why.

First of all, this type of research methodology addresses learning as a social process, i.e. learning is not considered as something that takes place purely in the learners' head but as a process of interaction between the learner and the environment (co-learners, artefacts, culture, teacher, etc) and emphasis is placed on this interaction. This approach is highly appropriate for ReMath Project, considering that our aim is to describe the role of technology in learning and design technologies that could bring about new forms of learning *within* school environment.

Furthermore, one of the objectives of WP4 is to provide validation of each DDA in respect to its functioning as didactical tools and the consistency of such functioning in relation to the theoretical assumptions, on which the DDA and its use have been designed. Hence we need to put under investigation the theoretical assumptions underpinning both the design of a DDA and of its use. In particular, these assumptions are assumed to inspire the design of the Pedagogical Plans centred on the use of the DDAs and developed within WP3. Consistently with the teaching experiment methodology, we will question the soundness of these assumptions through the implementation of the Pedagogical Plans in "real" teaching/learning settings.

Finally, in order to attain the objective of providing new insight on a) means of using technologies to support learning b) design issues concerning both DDA and Pedagogical Plans and c) learning processes in relation to the use of technologies, we expect to receive important feedbacks at the theoretical level from the experimentation. This objective is consistent with one of the main features of teaching experiments, that is to test the validity of the hypotheses driving teaching interventions, to favour their refinement and to generate new hypotheses in an iterative process.

I.2. Cross-Experimentation

From a methodological perspective, the cross-experimentation was planned to compare each team's experimentation with respect to the use of a theoretical framework different from the one in which the design of the DDA was rooted. In particular, the cross-experimentation aimed at (Artigue et al., 2007b):

- Understanding what it is implied when "tuning" the use of a DDA to the specific pedagogical aims and research objectives of a team that has not developed it.

- Understanding similarities and differences in the educational context set up by each team to experiment a DDA.

- Understanding/discovering implicit aspects embedded in the used DDAs.

The relationship between theoretical reflection and cases of practice is one of the main issues that characterised the effectiveness of the cross experiment both as a tool for comparing/integrating research approaches, and as a tool for investigating how to employ DDAs in mathematics education.

In particular, researchers involved in the cross experimentation witnessed the importance of the request to conduct an explicit reflection on the relationships among different elements of a teaching experiments: the formulation of research questions, the adoption of theoretical frameworks, the definition of educational goals, and the analysis of DDA tools.

I.3. A Minimal Theoretical Framework and a Common Research Question

As far as the Cross Experimentation is concerned, the main issue we were asked to face was that of articulating the local level of each Teaching Experiment and the global level of the research project. In fact, each team's local contribution is expected to address the Project general issues from the perspective the team is more familiar with. Those contributions are expected to focus on representation, and would hence have *per se* a common base. However the risk of a fragmentation of the different results has been always considered possible and we intended to control it as much as we can.

For that purpose, we designed two instruments for acquiring information on the planned local Teaching Experiments: a "Protocol of experimentation" and a "Profile of experimentation". Protocols had the goal of describing the main characteristics of the Teaching Experiments,

and Profiles, directly inspired by the ITF, had the goal of pointing out the principal concerns of each of them.

Our reflection on Protocols and Profiles led to the conclusion that *sharing research questions* would have been a potentially effective tool to limit fragmentation and foster the possibilities of comparing and integrating the single Teaching Experiments results Obviously, for the purposes of integrating and comparing the different results, the need to develop and share a *common language* became crucial as well.

As global result of our reflection, we agreed on some main assumptions, which we took as the skeleton of our implicitly shared theoretical framework concerning Representation:

1. No direct access to mathematical objects is possible, rather mathematical meanings are represented through language, formal mathematical notations, informal idiosyncratic representations, ...

2. Representations play a fundamental role in the "generation" of mathematical meanings, and this role is assumed to be crucial in the teaching/learning of mathematics.

3. Digital artefacts can provide representations of mathematical objects with a clear potential of generating mathematical meanings.

We also converged on the idea that the notion of representation could be made more precise through the following description. A **representation** can be seen as

> *a relationship between:*
>
> - *a **representing**, i.e. something with a perceivable nature, accessible to one's senses, and*
>
> - *a **represented**, i.e. something which is not accessible to one's senses but which is considered as existing in some sense (e.g. an idea, a concept, a process...).*
>
> *A relationship does not exist in itself; it is such only from someone's perspective. That is to say that a representation is such only if there is **someone** who recognizes a relation between something perceivable – a **representing** – and a corresponding **represented**.*

The combination of the main assumption and of the outlined description constitutes what we call a *Minimal Theoretical Framework* (MTF) on representation.

This perspective taken on representation entails that individuals can share *representations* only through sharing the perception of the perceivable *representing*. However, it may happen that in spite of common perception of the *representing*, they fail to share the *represented*.

Starting from the MTF on representation as stated above, we decided to outline the following **Common Research Question** (CRQ) to be addressed in our experimentation:

> *How can the representations identifiable in the DDAs be put in relationship with the achievement of specific educational goals?[1]*

I.3.1 From the CRQ to the articulation in the Teaching Experiments

For each Teaching Experiment it carries out, each team was invited to reformulate the Common Research Question according to the specific Theoretical Framework assumed to frame its own investigation. The idea was that the different theoretical frameworks should provide means to refine the Common Research Question, articulating the general common issues according to different theoretical tools, but preserving its essence, expressed in the Minimal Theoretical Framework. The Minimal Theoretical Framework is also expected to provide a means of articulating the different answers given at level of local experimentations.

I.4 The Teaching Experiment Guidelines

After the discussion of the MTF, we set up a new version of the Teaching Experiment Guidelines. The rationale of this document was inspired by the intention of addressing the three dimensions through which WP4 is expected to contribute to the Project (see Introduction):

1 providing "the validation of the D.D.A.'s both in respect to their functioning as didactical tools and the consistency of such functioning in relation to the theoretical assumptions, on which scenarios (tools and activities) have been designed" (ReMath Project Proposal part B, p. 59);

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[1] Elaborating on the previous definition of representation, the following *legenda* can be provided:
representations: by representation we mean a relationship between a **representing** , i.e. something with a perceivable nature, accessible to one's senses, and a **represented**, i.e. something which is not accessible to one's senses but which is considered as existing in some sense (e.g. an idea, a concept, …)
identifiable: a representation does not exist in itself, it is such only from someone's perspective. That is to say that a representation is such only if there is **someone** who recognizes *a relation between something perceivable – a representing – and a corresponding represented*. Thus when speaking of representations we should specify not only representing and represented but also the perspective according to whom they are related. As a consequence one might need to distinguish among the perspective of the designers, of the experimenters and of the students involved in the experimentation and possibly to compare them.
relationship: we use this vague term purposefulle to refer to the link between the identified representations and the stated educational goals: it may encompass the (semiotic, cognitive, institutional…) processes which underlay the achievement of the EGs. In particular, this relationship amy refer also to the relationship between representations identified by the experimenters and those "constructed by", "generated by", or "ascribable to" students. Of course, in order to specify and make such relationship clear the analysis of the modalities of use of the DDAs may b highly helpful.
Specific educational goals: here we are assuming that DDAs are used in the experiments for achieving specific educational goals, which are not necessarily the final educational goals of the implemented pedagogical plan but are specific to the use of the DDA. In any case, it can be useful to make clear the relationship between the global educational goals of the teaching experiment and the goals specific to the use of the DDA.
How: it refers to the terms, the ways, or the criteria… according to which one can describem give account or attest… the relationship between the identified representations and the achievement of specific educational goals. The meaning of this "how", and of the term "relationship" as well, might be specified according to the assumed theoretical frameworks.

2 producing common results that enhance "our understanding of meaning-making through representing with digital media" (ibidem, p. 9);

3 "providing validation for the [integrative theoretical] framework" (ibidem, p. 8).

Obviously, such aims are not independent on each other. In the following we will try to specify these aims and, in so doing, to highlight the connections among them.

I.4.1. Providing the validation of the DDAs and their use, as designed in the Pedagogical Plans

Common features of the design of a DDA and of the design of its use within the frame of the Project were identified: designers have in mind some educational goals which can be achieved through the use of the DDA, and educational hypotheses are assumed to link the use of the DDA with the achievement of the educational goals envisaged.

For dealing efficiently with the issue of providing the validation of a DDA, we considered useful to distinguish between teaching experiments carried with *familiar* and *alien* DDAs. We assumed that the use of a familiar DDA would be designed consistently with the educational hypotheses inspiring the design of the DDA. Given this premise, the following couples of questions were addressed to be answered after the experiments:

DDA design

Have the educational goals envisaged in the design of the DDA been achieved in the teaching experiment?

Does the experimentation attest the consistency between the hypotheses made in the DDA design and the achievement of the educational goals?

Design of the use of a DDA

■ Have the educational goals envisaged in the design of the Pedagogical Plan and specific to the use of the DDA been achieved in the teaching experiment?

■ Does the experimentation attest the consistency between the hypotheses made in the PP design and the achievement of the educational goals?

The above questions are to be answered after the experiments, but in order to strengthen and compare the results of the experimentation it was decided that each team should specify, a priori, for each experiment:

· the educational goals envisaged

· the hypotheses made

· the "criteria" for attesting the consistency between the achievement of the educational goals envisaged and the hypotheses made.

Thus, keeping the distinction between DDA design and design of the use of a DDA, for each local teaching experiment the following information was asked to be given **a priori**:

Experiments centred on the use of a "familiar DDA"

(1) Specify at least one educational goal that you have envisaged when designing the DDA.

(2) Specify the hypotheses that you made when designing the DDA and that link the use of the DDA with the envisaged educational goals.

(3) Specify how (according to what criteria) you could attest (on the basis of the experiment) the consistency between the hypotheses made when designing the DDA and the achievement of the educational goals envisaged.

Experiments centred on the use of an "alien DDA"

(1) Specify at least one educational goal that you have envisaged when designing the Pedagogical Plan centred on the use of the DDA.

(2) Specify the hypotheses that you made when designing the PP and that link the use of the DDA with the envisaged educational goals.

(3) Specify how (according to what criteria) you could attest (on the basis of the experiment) the consistency between the hypotheses made when designing the PP and the achievement of the educational goals envisaged.

I.4.2. Producing common results that enhance "our understanding of meaning-making through representing with digital media"

As said above, it was decided that all the partners involved in experimentation should reformulate the CRQ in their own theoretical terms, preserving the essence of the CRQ itself, and address it through their local teaching experiments. This process is not meant to exhaust all our efforts for achieving the objectives of WP4, that is enhancing our understanding on a) means of using technologies to support learning, b) design issues and c) learning with technologies.

In order to compare such Specific Research Questions in an effective way, a common effort had to be made by all the partners to relate these Specific Research Questions to the general issues addressed within the Project, the main research-concerns and the theoretical frameworks.

For this purpose we refined the Profile of Experiment exactly with the twofold aim of:

(1) structuring the information and thus potentially facilitating the comparison of results at the end of the experiment, and

(2) supporting teams in their effort of making clear the various elements in focus.

The refinement of the Profile of Experiment gave birth to the *Research Profile of the Teaching Experiment*. Such Profile is meant to make it possible to isolate "*research concerns*" from other more general concerns that can inspire at large the teaching experiments.

Team agreed that, when filling the Research Profile, they should express only their research concerns. Consistently, each team should formulate Research Questions related to its own *top*

(rate 3) research concerns; the link among research questions and related concerns should be made clear as well.

We were not assuming the existence of a one-to-one correspondence among top research concerns and Research Questions.

however we expected that some of the *top research concerns* were related to the team reformulation of the Common Research Question – in fact the CRQ results from the analysis of the first Profiles produced by all the partners. On the other hand the same research concerns could also originate other (Specific) Research Questions. In addition more possible top research concerns could be expressed in the Research Profile: these concerns should result in (be related to) the formulation of Specific Research Questions.

We kept the same structure of the previous version, but we introduced important differences. For instance, we added the explicit request of analytically specifying the theoretical construct related to each concern, meant both to structure information and to support the effort of clarification.

I.4.3. Feeding the development of ITF

Because of its centrality in framing the Profile, and generally speaking in preparing the field for comparing the different Teaching Experiments, an implicit validation of the ITF is expected form the effectiveness of the comparison process.

More specifically, validation of the ITF is questioned at three different levels:

The level of the formulation of the CRQ

The level of integration of the different answers to the different formulations of the CRQ

The level of the specific research questions, i.e. how is it possible to compare and integrate the answers to the different SRQs.

We expected to highlight potentialities and limits of the ITF and, accordingly, suggest possible revision and refinement of the ITF itself.

I.5 Guidelines

The guidelines are constituted by five sections:

Identity. It is meant to collect basic information about the plan of the teaching experiment.

Validation of the DDA. It is meant to provide a common structure to locally address the issue of validating both the functioning of DDAs as didactical tools, and the consistency of such functioning with theoretical assumptions assumed in the design of the DDAs and of their use.

Common Research Questions. It contains a Common Research Question related to Representation, which each team has to reformulate in its own theoretical terms and accordingly to address in its experimentation. This Common Research Question is formulated

in the terms of the Minimal Theoretical Framework, and it has been identified as a result of the analysis of the first Profiles of Experimentation provided by all the partners.

Research Profile of the Teaching Experiment. It is meant to support the teams to make explicit their research concerns, in respect to each teaching experiment. Each team should formulate and address Research Questions related to the research concerns to which the team gives high priority (top research concerns). These Research Questions can include the teams' reformulation of the Common Research Question or Specific Research Questions as well.

Specific Research Questions. It is meant to collect the Specific Research Questions which each team wants possibly to address (in addition to the questions in section "Validation…" and "Common…"). The relation among Specific Research Questions and top research concerns should be made explicit.

In order to compare, and possibly integrate the results of the different Teaching Experiments in an effective way, for each Research Question addressed (is it the reformulation of the CRQ or a SRQ) the teams are asked to specify:

- the top research concerns to which the Research Question is related;

- the key-elements of their own theoretical frames needed to formulate the Research Question itself;

- what kind of data are expected to contribute to give answer to the RQ and how;

- whether the choice of the kind of data is related to the adopted theoretical frameworks.

The Teaching Experiment Guidelines can be found in Annex.

II. Methodological tools for the Teaching Experiments analysis

The Teaching Experiment Guidelines was intended as a methodological tool for:

- supporting teams in their effort of making clear the various elements in focus in their respective Teaching Experiments, and

- structuring the information and thus potentially facilitating the comparison of results at the end of the experiments.

Following the Teaching Experiment Guidelines, each team produced a Teaching Experiment Portrait for each Teaching Experiment it developed. In particular, those documents contain: (a) the specification of the educational goals of the Pedagogical Plans implemented, as well as of the didactical hypotheses underpinning them, and (b) the formulation of the different Research Questions (both the Reformulation of the Common Research Questions and the possible Specific ones) addressed through the Teaching Experiments.

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Since the very beginning we started to discuss how structuring the presentation of the results of our respective Teaching Experiment so as to facilitate the comparison of the results themselves. There was the need of both referring to a common language and using a common format for communicating the emerging results. we decided to adopt the language emerging from the construct of Didactical Functionality, the notion of Representation, the Minimal Theoretical Framework and the Integrative Theoretical Framework. we wanted to communicate our results and convey ideas in a way to facilitate our reciprocal understanding, without renouncing to our own theoretical frames, not losing the richness of difference. Thus we developed a shared format for presenting our results. All that brought to the set up of two distinct methodological tools: the Teaching Experiment Synthesis Frame, and the Teaching Experiment Analysis Guidelines.

The former is meant to give a common structure for preparing short accounts summarizing both the main results and the evidence supporting those results. Short summaries of the TEs are also expected to provide that context of information needed to make sense of the other teams' answers to their own Research Questions.

The latter provides a common structure to frame the answers to the Research Questions addressed through the TEs and formulated in the TE Portraits. A detailed description will be given in the next section.

> **Teaching Experiment Synthesis Frame**
>
> 1. A section about students' achievements
>
>   1a. achievements with respect to the a-priori envisaged ed. goals.
>
>   1b. specification of the evidence supporting the claimed achievements
>
> 2. A section discussing the relation between students' achievements and the use of the DDA in the context of the PP. In this section the issue of 'representation' should be addressed, according the different theoretical approaches that each team adopt

3. A section addressing the issue of the relationship between what envisaged when planning the PPs and the actual results of the TE:

students' achievements are consistent with what we envisaged a priori, because... or,

the actual findings of the TE were not envisaged.... a possible explanation might be... or,

we did not find what we envisaged... a possible explanation might be...

Worksheet 1. Teaching Experiment Synthesis Frame

II.1 Teaching Experiment Analysis Guidelines

The Teaching Experiment Analysis Guidelines is meant to provide a common structure to frame the answers to the Research Questions addressed through the TEs and formulated in the TE Portraits.

The structure of the Teaching Experiment Analysis Guidelines mirrors at some extent the structure of the Teaching Experiment Guidelines. It is constituted by three sections addressing respectively: the evaluation of the implementation of the designed PPs from the point of view of their didactical effectiveness, the answers to the different Reformulations of the Common Research Question, and the answers to the possible Specific Research Questions investigated through the TEs.

II.1.1. DDA&PP Evaluation

As recalled in the introduction (§Introduction), the Empirical Research coordinated within WP4 is intended to contribute to the Project objectives in different ways. The first section of the TEAG addresses the issues of providing feedbacks on the DDAs functioning as didactical tools and evaluating the consistency of such functioning in relation to the theoretical assumptions, on which PPs have been designed. In fact, explicit assumptions within our Project are that: (i) DDA designers and PP designers as well, have in mind some educational goals which can be achieved through the use of the DDA, and (ii) educational hypotheses are assumed to link the use of the DDA with the achievement of the educational goals envisaged.

Moreover, we think that the functioning of a DDA cannot be evaluated in itself without considering how the DDA is used. The achievement of specific educational goals has to be evaluated considering the whole Pedagogical Plan designed around the use of such DDA and the effect of the general context within which the pedagogical Plan is implemented. Consequently one cannot separate the evaluation of the didactical functioning of a DDA from the evaluation of the PP designing the use of that DDA: feedbacks on the didactical functioning of a DDA are also feedbacks on the PP and vice versa. For that reason we speak of DDA&PP evaluation as a single (certainly complex but not reducible) process.

Before the experimentation, each team was asked to:

a. select a-priori some educational goals linked to the use of the DDA and envisaged when designing the PP;

b. specify the educational hypotheses, made when designing the PP, and linking the use of the DDA with the envisaged educational goals; and

c. specify how it is possible to attest (on the basis of the experiment) the consistency between those hypotheses and the achievement of the educational goals envisaged.

In the analysis of their own TEs, the experimenting teams are asked to reconsider those educational goals and hypotheses, and to evaluate whether they can attest the achievement of the envisaged educational goals and the consistency of the educational hypotheses.

With that respect, we remark that because of the very nature of a teaching experiment – the complexity of the system, the high number and low controllability of the variables – in the case that an experiment fails to attest the consistency between educational hypotheses and the achievement of educational goals, this does not mean that such consistency does not hold.

Hereafter, we reproduce the corresponding section of the TEAG

Validation of DDAs and PPs

In this section, we refer to the Educational Goals and Hypotheses specified in the corresponding section of your own TE Portrait. The same questions apply to both TEs based on a Familiar DDA and to those based on an Alien one.

Familiar/Alien DDA:

> (1) WERE THE EDUCATIONAL GOAL(S), SPECIFIED IN YOUR TE PORTRAITS, ACHIEVED? HOW CAN YOU ATTEST THAT?

> (2) ON THE BASIS OF YOUR EXPERIMENTATION, CAN YOU CONFIRM THE SOUNDNESS OF THE HYPOTHESES SPECIFIED IN YOUR TE PORTRAIT, AND THE RELATIONHIP WITH THE ACHIEVEMENT OF THE EDUCATIONAL GOALS?
>
> EXPLAIN BY MAKING REFERENCE, IF POSSIBLE, TO THE CRITERIA SPECIFIED A-PRIORI IN YOUR TE PORTRAIT.

Worksheet 2. TEAG: DDA&PP evaluation

II.1.2. Reformulated Common Research Questions

As a result of our reflection on the setting up of the global experimental design, there was the recognition of the prominent role of a clear formulation of research questions. Firstly, research questions inform local experimental designs, which have to be to provide answers to such questions, and then as linking tool between the different experimentations. That led to the conclusion that *sharing research questions* would have been a potentially effective tool to

limit fragmentation and foster the possibilities of comparing and integrating the single Teaching Experiments results so as to allow producing common results common results enhancing our understanding of meaning-making through representing with digital media.

In section §I.3 we outlined the process carried out to develop a methodological tool to address the issue of producing common results. Such tool was centred in the formulation of the Minimal Theoretical Framework and the related Common Research Question that we remind for the reader's convenience:

> *How can the representations identifiable in the DDAs be put in relationship with the achievement of specific educational goals?*

All the experimenting teams reformulated the CRQ in their own theoretical terms, preserving the essence of the CRQ itself, and addressed it through their local Teaching Experiments. They also specified which *top research concerns* were related to their reformulation of the Common Research Question. Those Reformulations of the CRQ and the related concerns ware specified in the TE Profiles.

The analysis of the TE should provide answers to the corresponding Re-CRQ. It was agreed that such answers should be expressed in the terms and according to the language provided by the Theoretical Framework which each team adopted. In order to facilitate comparisons among the results, teams were asked to:

i. articulate the answers making reference to our developed common language, namely: the construct of DF, the MTF and the language of concerns;

ii. try to distinguish as far as possible between observation and elements of observation and their interpretation;

iii. specify which concerns drive the analysis of the TE itself and how.

Common Research Question

This section is meant to collect your answers to your Reformulation of the CRQ (ReCRQ), contained in the TE Portrait.

CRQ.

> *How can the representations identifiable in the DDAs be put in relationship with the achievement of specific educational goals?*

---

1. REPORT YOUR RE-FORMULATION OF THE COMMON RESEARCH QUESTION (RE-CRQ)

---

2. ANSWER YOUR RE-CRQ.

   WITHOUT RENOUNCING TO YOUR OWN THEORETICAL FRAMEWORK(S) AND LANGUAGE, TRY TO ARTICULATE YOUR ANSWER BY MAKING REFERENCE TO THE

THREE POLES OF THE NOTION OF DIDACTICAL FUNCTIONALITY AND TO THE SHARED MINIMAL THEORETICAL FRAMEWORK.

3. SPECIFY:

   - THE KIND OF DATA YOU ANALYSED;

   - THE SPECIFIC ELEMENTS OF OBSERVATION.

4. DESCRIBE HOW THE ELEMENTS OF OBSERVATION WERE USED TO SUPPORT YOUR ANSWER TO THE RE-CRQ.

   IF POSSIBLE, MAKE EXPLICIT WHICH ELEMENTS OF YOUR THEORETICAL FRAMEWORK(S) WERE USED IN THE ANALYSIS PROCESS AND HOW.

5. MAKE EXPLICIT WHICH CONCERNS GUIDED YOUR ANALYSIS PROCESS AND HOW.

Worksheet 3. TEAG: Re-CRQ section

II.1.3. *Specific Research Questions*

When designing their Teaching Experiments, some teams decided not to address only the Common Research Questions, but raised more questions (named Specific Research Questions) linked to the issues addressed within the Project, for example, means of using technologies to support learning, design issues and learning with technologies.

Obviously we are also interested in investigating the possibility of comparing the answers to these Specific Research Questions addressed independently in each experiments and hopefully drawing common results.

In fact, the Teaching Experiment Analysis Guidelines contain a section for gathering the answers to the Specific Research Questions. The structure of that section mirrors the structure of the previous section, with the addition that teams are asked to make explicit possible link between the addressed SRQs and their Re-CRQ.

Specific Research Questions

This section is meant to collect your answers to your SRQ, contained in the TE Portrait.

As one can notice, we are proposing the (more or less) same frame to articulate both the answers to the ReCRQ and those to the SRQs.

For each possible SRQ fill the following.

1. REPORT YOUR SRQ.

2. ANSWER YOUR SRQ.

   WITHOUT RENOUNCING TO YOUR OWN THEORETICAL FRAMEWORK(S) AND LANGUAGE, TRY TO ARTICULATE YOUR ANSWER BY MAKING REFERENCE TO THE THREE POLES OF THE NOTION OF DIDACTICAL FUNCTIONALITY AND TO THE SHARED MINIMAL THEORETICAL FRAMEWORK.

3. SPECIFY:
   - THE KIND OF DATA YOU ANALYSED;
   - THE SPECIFIC ELEMENTS OF OBSERVATION.

4. DESCRIBE HOW THE ELEMENTS OF OBSERVATION WERE USED TO SUPPORT YOUR ANSWER TO YOUR SRQ.

   IF POSSIBLE, MAKE EXPLICIT WHICH ELEMENTS OF YOUR THEORETICAL FRAMEWORK(S) WERE USED IN THE ANALYSIS PROCESS AND HOW.

5. MAKE EXPLICIT WHICH CONCERNS GUIDED YOUR ANALYSIS PROCESS AND HOW.

6. IS YOUR SRQ MEANT TO CONTRIBUTE TO PROVIDE AN ANSWER TO YOUR RE-CRQ? IF YES, HOW?

Worksheet 4. TEAG: SRQ section

III. On the actual implementation of the TEs

In this chapter we report on the actual implementation of the TEs. The information provided was collected through a questionnaire, distributed and filled in September-October 2008. The questionnaire aimed at gathering:

- quantitative information concerning the implementation of the TEs (number of classes involved, school grade, number of school-hours), and

- synthetic qualitative information about possible deviations from the original planned TEs. Each team was asked to report on the possible variations of the original planned TEs which occurred during the implementation of the TEs. More in details, teams were asked to distinguish between minor and major variations, and specify whether and how those variations affected the possibility to answer the TE Research Questions.

The table below is meant to summarize the questionnaire answers of each TE (the full answers can be found in the Annex).

	No. of classes	Grade	Age	No of school-hrs	Period	Variations	Influences
Alnuset ITD (familiar)	1	10	15	20	Oct-Dec07	Minor	No
Alnuset MeTAH (alien)	1	10	15-16	3	Apr07	Minor	No
Aplusix MeTAH (familiar)	3	9	14-15	4	Nov-Dec07	Major	No
		10	15-16	3 and 2.25[2]	Sept-Oct07	Major	
		10	15-16	4	Dec07	Minor	
Aplusix ITD (alien)	1	7	12-13	8 (+2)[3]	Nov-Dec07	No	
Aplusix Unisi (alien)	2	9	14	18	Oct07-Jan08	Minor	No
		9	14	20	Oct07-Jan08	Minor	
Casyopée	2	11	17	10	Oct-Dec07	No	

---

[2] The class was splitted in two groups involved respectively in a 2.25 hours and in a 3 hours experimentation.
[3] The enactment of the TE was structured in 8 hours devoted to teaching sessions and 2 hours devoted to a pre-test and a post-test session. The distinction is made by ITD team.

Didirem (familiar)		11	17	10	Oct-Dec07	No	
Casyopée Unisi (alien)	4	12	17-18	12	Oct-Dec07	Minor	No
		12	17-18	13	Oct-Dec07	Minor	
		12	17-18	8	Oct-Dec07	Mayor	
		11	16-17	11	Mar-May08	Minor	
Cruislet ETL (familiar)	2	11	15-16	20	Oct07-Apr08	Minor	Answering the SRQ
		11	15-16	8	Oct-Dec07	Minor	
Cruislet Didirem (alien)	2	11	17	5	Sept-Oct07	Major	Ed.Goals, hypotheses and CRQ
		11	17	5	Sept-Oct07	Major	
MaLT ETL (familiar)	1	7	13	18	Nov-Dec07	Minor	No
MaLT IoE (alien)	1	8	12-13	8	Nov07	Minor	No
MoPiX IoE (familiar)	1		16-19	15	Oct-Dec07	Minor	Answer to CRQ
MoPiX ETL (alien)	1	12	17-21	25	Nov-Dec07	Minor	No

Summary	22		248	No: 3	No: 10
				Minor: 14	Yes: 3
				Mayor:5	

Table 2. TEs implementation: Questionnaire answers

III.1 Quantitative analysis

Considering the qualitative nature of the methodology adopted for empirical research, the number of classes involved (22) and the overall amount of school hours (248) give a clear global picture of the efforts lavished by the WP4 partners. The weight of this effort suits the

importance of WP4 in connection with the other WPs: in fact empirical research is expected to provide with feedbacks concerning the DDA design and development (WP2), the PP design and implementation (WP3), and the development of the ITF (WP1).

Besides the number of classes and the global amount of school hours, more elements can be focused on in order to get an overall view of the range of the WP4 empirical research.

III.1.1 School levels

A first element concerns the school level of the classes involved. The TEs carried out involved, all together, classes of a wide range of school levels: since 7th to 12th grade (students' age 12 to 21). The diagram in Figure 1 shows the number of classes for each grade.
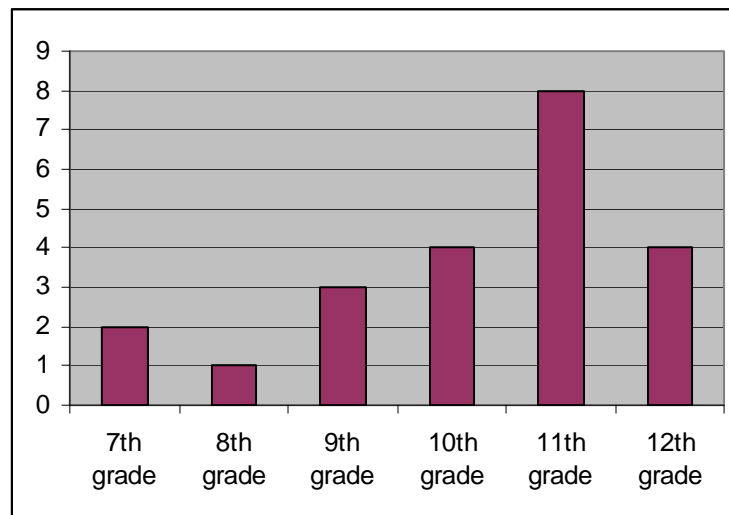
Figure 1 No. of classes per grade

From the diagram it appears that the large majority of TE implementations took place in grade 11 classes (8 over 22). That does not mean that the majority of TEs was designed for grade 11, rather that occurs because the different number of implementations of each TE: for instance, the IoE TE with MaLT was implemented just in one class, while Unisi TE with Casyopée was implemented in four classes.

In order to avoid these kinds of repetitions, we might take for each TE the approximated average of the school levels of the classes involved in the experimentation (for the IoE TE with MaLT this value is 8, while for the Unisi TE with Casyopée this value is 12, being 11.75 the average value). The diagram in Figure 2 displays this kind of information per grade. We do not consider this last one a more valuable indicator of the range of our experimentation; it is simply a different indicator which may contribute to provide a more precise, though synthetic, overview of our work.
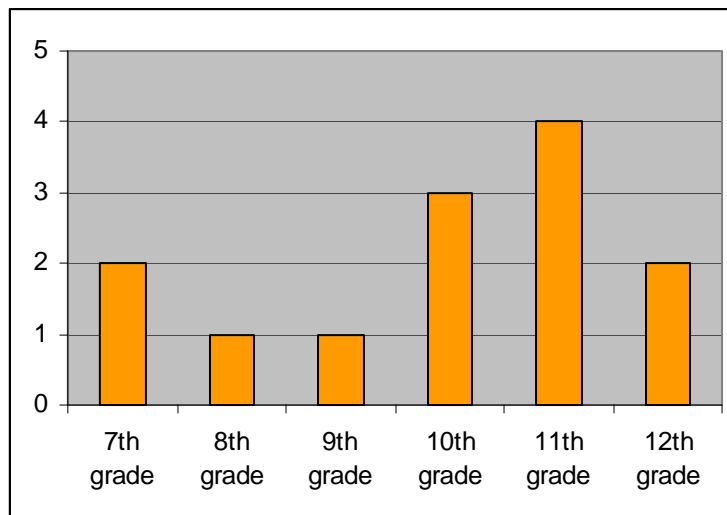
Figure 2 "Average" No. of classes per grade

III.1.2 Number of school hours

Another interesting aspect is the number of school hours used for the implementation of the TEs. Table 2 shows a significant diversity among the TEs: from 2.25 to 25 hours. A less dispersive view can be obtained by grouping the TEs according to the number of school hours used. We selected the following groups: 1 up to 5, 6 up to 10, 11 up to 15, 16 up to 20, and 21 up to 25. The resulting picture is shown in the diagram below (Figure 3).
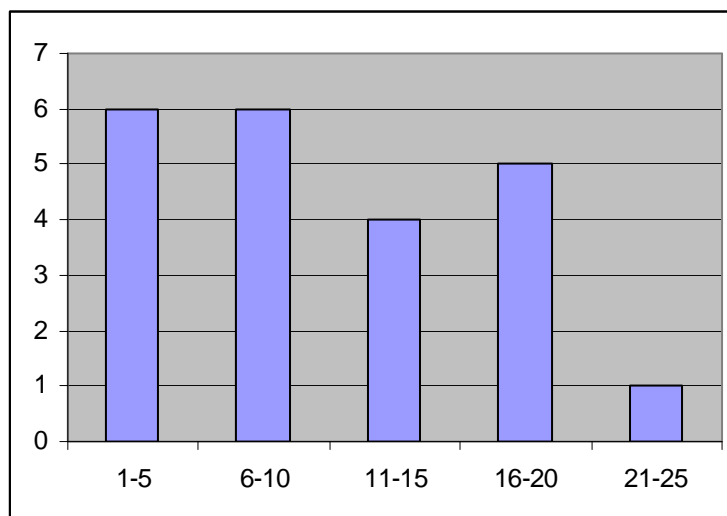
Figure 3 No. of classes per group of hours

This diagram presents the same disadvantages as the previous one showing the number of classes per school level (Figure 1): in a sense there are "repetitions" due to the different number of classes where different TEs were implemented. Once again, in order to avoid these repetitions we can take for each TE the approximated average of the number of school hours used in the implementation of a same TE (just to keep the same example: for the IoE TE with MaLT this value is 8, while for the Unisi TE with Casyopée this value is 11). The resulting "average" number of classes for group of hours is depicted in the diagram (Figure 4).
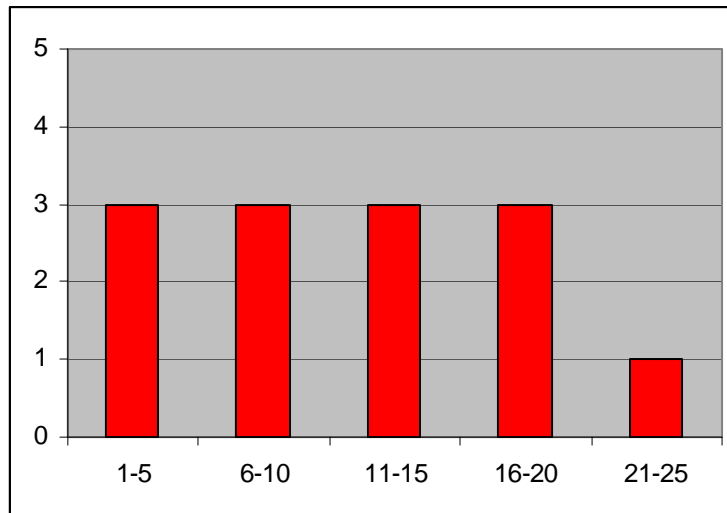
Figure 4 "Average" No. of classes per group of hours

III.1.3 Number of classes and number of school hours per DDA

Finally, it is interesting to compare, DDA by DDA, the number of classes and the number of school hours used in the implementation of the TEs. The diagram in Figure 5 summarizes those data. One can notice that each DDA was experimented for not less than 20 school hours (being 23 the minimum). Aplusix and Casyopée are the DDAs with the largest number of school hours used in implementation (57 and 64 respectively). That appears to be consistent with the number of classes involved in the experimentation (6 for both the DDAs).

	Alnuset	Aplusix	Casyopée	Cruislet	MaLT	MoPix
☐ No. of classes	2	6	6	4	2	2
■ No. of hours	23	57	64	38	26	40
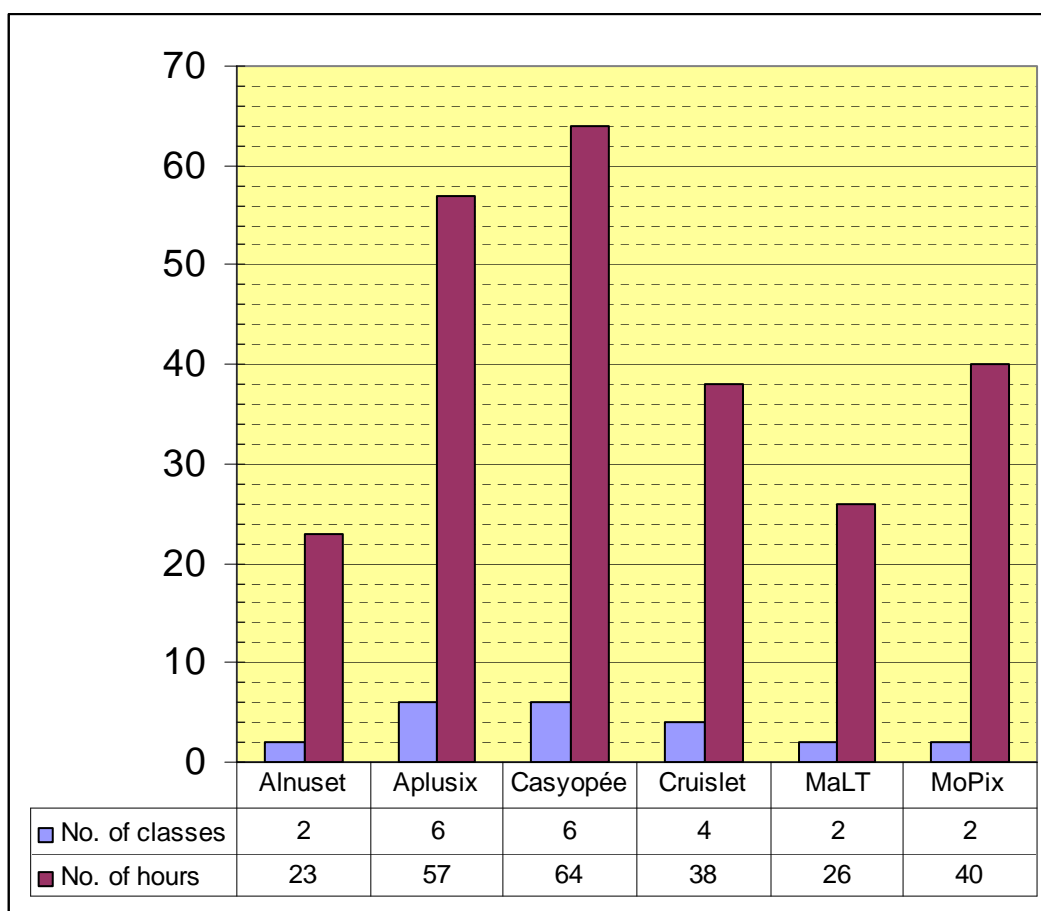
Figure 5 No. of classes and hours per DDA

III.2 Qualitative analysis

Together with the quantitative information summarized and commented in the previous sections, the questionnaire allowed to collect qualitative information about possible deviations of the actual implementations of the TEs from the original plans.

Table 2 shows that 2 TEs were implemented in perfect accordance with the original designs of the TEs: the ITD TE with Aplusix, the implementation of which involved one class, and the Didirem TE with Casyopée, the implementation of which involved two classes.

Minor changes in the implementation

Most of the TEs underwent minor changes, small adjustments, during their implementation (14 out of 22 implementations). Because small adjustments were considered inevitable at some extent, teams were not asked to specify those adjustments (in order to avoid an overabundance of information difficult to manage). Anyway there are some teams who describe the changes made. According to teams' declarations, those changes concerned mainly the time-schedule of the teaching intervention, and some variations in the activities proposed.

The modifications made did not nearly affect the possibility of the teams to address the research issues which they meant to investigate through their TEs. Actually there is only one exception: IoE TE with MoPiX.

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"More time than anticipated was spent on some aspects of the teaching sequence. As a consequence most students did not complete tasks related explicitly to graphing or interactions between objects.

The omission of work on graphs prevented us from addressing one of the sub-questions to our reformulation of the CRQ which referred specifically to graphical representation."

(IoE TE with MoPiX: questionnaire on the TE implementation)

Major changes in the implementation

Major variations concern the implementation of 3 TEs in 5 classes: implementation of MeTAH TE with Aplusix in 2 classes, implementation of Unisi TE with Casyopée in one class, implementation of Didirem TE with Cruislet.

It is interesting to remark that these deviations are not limited to TE involving alien DDA. On the contrary, major deviations occurred in 2 classes (out of 3) where MeTAH team implemented their TE with Aplusix (familiar DDA).

From Table 2, it clearly emerges that major variations do not necessarily affect the possibility of answering the *a-priori* formulated RQs: MeTAH TE with Aplusix and Unisi TE with Casyopée. Conversely, as we already noticed, even minor variations can affect how a team could produce answers to the RQs at stake (see IoE TE with MoPiX)

In all the classes, changes concerned shortening of the PP to be implemented or modifications of the activities.

As for the Unisi TE with Casyopée, one of the experimenting teachers wished to use Casyopée but was concerned about the planned number of hours (11-12 school hours). The Unisi team together with the teacher agreed to shorten the PP (8 school hours), omitting the last two sessions.

However this decision did not affect the possibility to address the research issues stated a priori because it was possible to implement the omitted part in 3 other classes. Thus Unisi gathered enough data to answer the RQs in focus.

Also MeTAH team had to implement in 2 classes a shortened and modified version of their TE with Aplusix.

"Shortening the scenario:

- Conversion tasks RNL → RT et RU → RT were worked out with Aplusix in controlled mode only (initially, we planned to propose the same kind of activities in free mode as well) ;

- Conversion tasks RT → RNL were assigned as a homework (they were planned as classroom activities) ;

- Treatment tasks in RT were not compulsory. Teachers were free not to propose them, or propose them only to students with difficulties (Grade 10 teacher can benefit from one hour a week that can be dedicated to remedial activities with a small number of students having difficulties in math)."

(MeTAH TE with Aplusix: questionnaire on the TE implementation)

Anyway, MeTAH team declared that those modifications would not have affected the possibility to answer the RQs formulated *a-priori*.

Things went differently for Didirem TE with Cruislet. In fact the teaching sequence was designed in two phases: the former phase was compulsory and was devoted to familiarization with Cruislet, the latter one was appointed for using Cruislet within the frame of "travaux personnels encadrés", TPE, i.e. personal project work with the support of teachers. This second part was optional and no student chose it.

In summary, on the one hand deviations concerned the revision and re-design of tasks; on the other hand deviation consisted in the impossibility of implementing the second phase because students did not choose to carry out their personal project work with Cruislet.

> "The experiment went differently from what was planned in the guidelines. Thus it is necessary to adapt the goals, hypothesis and research question."
>
> (Didirem TE with Cruislet: questionnaire on the TE implementation)

After the indications coming from that experience, Didirem designed and carried out a new TE.

III.3 On context

The discussion of the deviations of the actual implementations of the TEs from the original plans raises the need of discussing the role of context in the planning and implementing the TEs.

The issue of context is surely an important and complex issue: on the one hand the notion itself of context is somewhat evasive; on the other one, there is not a single context for the actors and the activities involved in the design and implementation of a TE, but a multiplicity of contexts: i.e. the context of DDA development and distribution, the institutional and the local context in which the teams carry out their research.

Within the WP4, the teams agreed to face that issue by addressing a specific aspect. The emerging intention was to investigate which elements of the context of the teaching interventions raised the greatest problems for the teams in the design and the implementation of the teaching intervention themselves.

We are conscious that it is not easy to single out contextual elements and that sometimes stressing contextual issues can be perceived as taking a defensive position and weakening the research soundness. Nevertheless we think that an explicit reflection on how context affected the implementation of our TEs is especially important within our project.

In order to carry out such an investigation, the following questionnaire was set up.

THE ROLE OF CONTEXT IN THE PLANNING AND IMPLEMENTATION OF THE TEACHING INTERVENTION

1. (**Planning - Global**) Which elements of the global context raised the greatest problems or concerns in the planning of your teaching intervention? Explain in what sense, and how you coped with them.

2. (**Planning - Local**) Which elements of the local context raised the greatest problems or concerns in the planning of your teaching intervention? Explain in what sense, and how you coped with them.

---

3. **(Implementation - Global)** Which elements of the global context raised the greatest problems or concerns in the actual implementation of your teaching intervention? Explain in what sense, and how you coped with them. Explain In particular issues which were unexpected.

4. **(Implementation - Local)** Which elements of the local context raised the greatest problems in the actual implementation of your teaching intervention? Explain in what sense, and how you coped with them. Explain In particular issues which were unexpected.

5. **(Teacher)** Did (and possibly how) the fact that the teacher was/was not a member of the research team affect the implementation of the teaching intervention?

Worksheet 5. Questionnaire on context

The first four questions refer to the (envisaged and actual) context of implementation of the teaching intervention. After Del1, we distinguish between global (or institutional/cultural) and local (or situational) context. The global or institutional/cultural context includes elements such as: the national school systems, curricula, kinds of school (scientific school, technical institutes …), school grades, and so on. The local (or situational) context includes elements such as the specific teachers and classes that have been involved, school equipment, generic unexpected events, and so on.

The last question addresses whether and how the fact that a teacher is/is not member of the research team might affect the implementation of the teaching intervention. In that respect the teacher and her professional development are at some extent considered as an element of the local context, upon which an explicit reflection is needed.

In the following sections, we provide with a synthesis of the teams' answers to the questionnaire. The reader will find that some contextual factors (i.e. curriculum, school equipment, factors related to teachers in different ways) appear at different levels (both global and local) and with respect to both planning and implementing of the TE. That should not surprise, as we will try to clarify below there is a variety of complex factors can be related to curriculum, teachers' professional development or personal view, and so on, and such factors can arise and be inflected in different forms.

III.3.1 The consideration of the context in planning the teaching intervention

<u>Global context</u>

Two factors caused the main concerns:

a. *The institutional entry in school*. WP4 partners in Greece and UK had to enter a long process for getting an official authorization to access the school and carry out their experimentation

> *"In general, it is difficult to get into the schools in the UK. One needs to go through a long process of legal issues and coordination with schools. The ethical consideration is also an issue within the educational system and within the Institute of Education/LKL as well."*
>
>   (IoE questionnaire answers, TE with MoPiX)

Overall, many teams met difficulties for entering in schools: one of them is the need of long and complex *iter* for accessing the schools. Another one is the difficulty of finding teachers

disposed to collaborate in the design and implementation of innovative teaching intervention (we will dwell on that later).

All that constitutes a real problem, which math educators have to cope with. Education researchers have not any official privileged and straight channel for carrying out teaching experimentation in real class situation. Carrying out empirical research is made possible only through personal contacts established between researchers and few volunteer willing teachers. One may suspect that this difficulty is one of the causes of the commonly recognized and denounced low impact of educational research on school actual practice.

b. *The curriculum.* The term "curriculum" can assume a wide range of different connotations (Kelly, 2004). It can be intended as a more or less detailed official list of topics well established by the Minister of Education, or it can be intended as non official system of topics, practices, and so on, which is not formally and explicitly outlined but is deeply rooted in the school practice. As such, the consideration of curricular issues impacted the planning of the teaching intervention in different ways. Teams had to deal with a low degree of ICT integration in the school, with too much constrained curricula, and with a too marked distance between the curriculum and the DDA potentialities.

> *"Although the national Curriculum suggests the use of computers concerning geometry very few teachers follow these suggestions in their teaching practice for three main reasons:*
> *- computer use for teaching mathematics is not officially part of the curriculum;*
> *- schools computer laboratories are usually occupied for the teaching of informatics and;*
> *- teacher training for the use of computers in the teaching of mathematics is rather limited."*
>
> (ETL questionnaire answers, TE with MaLT)

> *"Relationship to the National Curriculum, school schemes of work, examination pressures and the time available: while the system officially gives space for teachers to add enrichment activities, in practice the curriculum is very constrained and many teachers and schools are unwilling to deviate from standard schemes of work."*
>
> (IoE questionnaire answers, TE with MoPiX)

> *"We had concerns about the difficulty of bringing activities not directly consistent with the curriculum, in the mathematics course (scientific stream) at 11th grade where the syllabus to cover is very heavy."*
>
> (Didirem questionnaire answers, TE with Cruislet)

In addition, Didirem had to face some problems arising from the system requirements for installing and using Cruislet.

Local context

The questionnaires answers put into evidence the existence of several factors related to the local context which influenced the design of the teaching intervention.

a. *Availability of experimenting teachers.* We already mentioned that some teams met difficulties in finding teachers disposed to collaborate in the design and implementation of the teaching intervention. MeTAH had to significantly revise its PP.

> *Our biggest concern was finding teachers who would be willing to experiment with Aplusix-tree. Initially, we wished to experiment with Grade 7 or Grade 8 classes. At this*

level, working with trees would be more appropriate and would contribute to the learning of algebra. Unfortunately, only the teachers who are members of our research team agreed to implement a scenario involving a tree representation, but they had Grade 9 and Grade 10 students. For this reason, we had to design activities with the aim of remediation to students' difficulties.

<div align="right">(MeTAH questionnaire answers, TE with Aplusix)</div>

b. *Curriculum*. Teams had to deal with curriculum as an element of the global institutional context but also as an element of the local context. In fact, when planning their teaching interventions, teams had to confront themselves with (i) how curricula were put into practice in the specific schools or in the specific classrooms, and with (ii) how the teachers perceived the curricula, the proposed teaching intervention and the relationship between them (distance, harmony, rupture…), and (iii) how students perceived the relationship between curricula and implemented teaching intervention.

"The National Curriculum and school scheme of work: during our discussion with the teacher, the issue of making the teaching experience relevant to the curriculum was dominant. The plan had to be adapted to achieve that goal. We also had to schedule the teaching sessions to fit in with the school's planned order of topics."

<div align="right">(IoE questionnaire answers, TE with MaLT)</div>

"[…] the scenario was elaborated by the Master student in agreement with the teacher of the experimental class. Therefore it was necessary to propose activities that could be easily integrated into the teacher's pedagogical sequence"

<div align="right">(MeTAH questionnaire answers, TE with Alnuset)</div>

Students' priorities and study loads: This concern was raised by the teachers since the students were studying for a high staked examination in Advanced Level Mathematics. It was necessary to present teachers and students with explicit links with the standard curriculum and to present the project to students as an activity that would support their learning in preparation for their examinations - and look good on their CVs when applying to university.

<div align="right">(IoE questionnaire answers, TE with MoPiX)</div>

The mentioned issue – how the teachers perceived the relationship between curricula and proposed teaching interventions – is clearly related also to the willingness of teachers to carry out experimentation in their classes.

c. *Time*. Another constraining aspect was time availability. Again one had to face the teachers' or school masters' perception of curricula and of how the planned teaching interventions fit them. In fact such perception directly influenced the amount of time teachers or school masters were disposed to devote for implementing the experimentation.

- *"Difficulties in fixing the time schedule, as we wanted too many hours for the 1st TE and the computer lab as well as the math teacher were not available.*
In order to overcome this, we used an additional 'after school' hour during the week."

<div align="right">(ETL questionnaire answers, TE with Cruislet)</div>

"The time available: The college and teachers were not prepared to allow us to use their scheduled teaching time for the teaching experiment. Students thus had to voluntarily give up some of their free time to be involved in the project. Finding a time slot to accommodate us was difficult, resulting in a smaller than anticipated number of participants."

<div align="right">(IoE questionnaire answers, TE with MoPiX)</div>

> *"The time available: Given the small amount of time available within the school's scheme of work, we had to schedule all sessions within a two week period and, moreover, had to make use of additional after school sessions."*
>
> (IoE questionnaire answers, TE with MaLT)

In the design phase IoE team had also to tune its teaching intervention according to the fact that the students were assessed as low achievers.

> *[…] the group of students we were able to work with were identified as low attainers. We reduced our expectations of what students would be able to achieve based on the school's assessment of their attainment level.*
>
> (IoE questionnaire answers, TE with MaLT)

Other factors raising difficulties in planning the teaching intervention were linked to technical equipment of schools: in particular the use of Cruislet (and possibly of Screen-Capturing software) was highly demanding for some school PCs system resources.

Synthesis

The following table synthesizes the elements of the global and of the local context which raised the greatest problems or concerns in the planning of the teaching interventions.

	Elements of the **global** context raising the greatest problems or concerns in **planning**	Elements of the **local** context raising the greatest problems or concerns in **planning**
Didirem Casyopée	Ø	Ø
Didirem Cruislet	Curriculum School ICT Resources	School ICT Resources
ETL Cruislet	Entry in school Curriculum	Time School ICT Resources
ETL MaLT	Entry in school Curriculum	Teachers' availability Student Population
ETL Mopix	Entry in school Curriculum	Ø
IoE MoPiX	Entry in school Curriculum	Curriculum Time
IoE MaLT	Entry in school Curriculum	Curriculum Time
ITD Alnuset	Ø	Ø
ITD Aplusix	Ø	Ø
MeTAH Aplusix	Ø	Teachers' availability

MeTAH Alnuset	Ø	Curriculum
Unisi Aplusix	Curriculum	Ø
Unisi Casyopée	Curriculum	Ø

Table 3. Contextual issues and planning of the teaching intervention

III.3.2 The consideration of the context in implementing the teaching intervention

Global context

Here again, the teams had to face different inflections of two already discussed contextual issues: curriculum and time.

a. *Curriculum*. Two teams point to curricular issues as affecting the implementation of their TEs: IoE, and MeTAH. The former had to face students' general perception of the distance between the official curriculum and implemented teaching intervention, in with respect to the examination system. That caused students' low attendance and lack of motivations. The latter experienced difficulties in integrating the teaching intervention in the established teaching practice shared within one of the schools involved in the experimentation

"The curriculum and examination system: These proved to be an ongoing issue, affecting student motivation and attendance. The college supported us by continuing to encourage students to attend."

(IoE questionnaire answers, TE with MoPiX)

"At the institutional level, for example, in the school where C2 class is, all Grade 10 classes progress in the math curriculum in the same way (same order, same rhythm). For this reason, it was very difficult for the teacher to integrate the whole scenario into the common sequence."

(MeTAH questionnaire answers, TE with Aplusix)

b. *Time*. Time constraints affected the implementation of IoE teaching intervention centred on the use of MaLT.

"The time available: as mentioned above, the main affect of the time was in putting the whole teaching experience only on 10 days with condensed schedule that added more pressure on students and affected their motivation and interests."

(IoE questionnaire answers, TE with MaLT)

Local context

a. *Student population.* The factors which mainly affected the implementation of the teaching interventions are related in a variety of ways to the student population. A first aspect concerns students' participation and regards two teaching interventions. Didirem teaching intervention with Cruislet was structured in two phases: the second one was appointed for using Cruislet within the frame of optional personal project work, and no student chose to participate. IoE teaching intervention with MoPiX involved only volunteers, and students' attendance was not stable throughout the experimentation.

Since participation in the teaching experience was voluntary, only seven students were involved. In some sessions attendance was reduced because of other events (e.g. attendance at interviews,

preparation for examination) that took priority for individual students. This had the consequence that we were unable to rely on stable groupings of students. The collaborative working aspect of our pedagogical plan was thus reduced and adapted.

(IoE questionnaire answers, TE with MoPiX)

A second aspect of student population relates to students' pre-existing knowledge. The implementation of both ETL and IoE teaching intervention was affected by students' lack or low experience in programming. In addition ETL had to cope with the fact that experimentation held in a multi-cultural school where many students did not speak Greek fluently.

"- Students' fluency in Greek. Many students of the multi-cultural school in which our experiment took place were not really fluent in communicating in Greek.
- Students limited experience with programming. Most of the students had not previous experience in programming with any language.
As a result the initial stages of our experimentation took more time than planned. However due to curriculum constrains the school was unable to offer more time for the experiment. This resulted in modifying/omitting the implementation of specific -initially planned- activities."

(ETL questionnaire answers, TE with MaLT)

Implementation of teaching intervention was also affected by the distance between students' envisaged and actual level of attainment. This regarded both IoE experimentation with MaLT (as mentioned in the previous section) and ETL experimentation with MoPiX.

As far as the Pedagogical Plan was concerned, we had to revise the time schedule twice and modify some of the activities. The reason for that was the fact the students felt that they needed more time to spend working on specific activities. Since we didn't want to impose our own pace of work, disregard the difficulties our students encountered and move on to the next activities, we decided to prolong the corresponding phases and give them more time to work on their models. This could not be considered to be a local problem as if this was an obstacle deriving from the students' specific characteristics, but it could be recorded as an unexpected problem that had as a result to ask the teachers for more school hours to complete the experimentation.

(ETL questionnaire answers, TE with MoPiX)

b. *Schoolrooms.* Some teams pointed to the inadequacy of school rooms for implementing the designed activities: for instance MeTAH denounced the impossibility of splitting the class into two groups, and IoE lamented the difficulties of letting students share computers, and consequently the difficulties of developing collaborative activities.

"The school's computing facilities: The computer laboratory in which our MaLT sessions were scheduled was arranged in such a way as to discourage group work. Students were also used to working individually when in this room. This meant that they were unwilling to share computers and there was less collaboration and discussion than we would have wished."

(IoE questionnaire answers, TE with MaLT)

"Material constraints led to different implementations of the scenario. For example, in the Grade 9 class it is not possible to split the class into two groups. Therefore, all sessions were done with the whole class, 2 students per computer and the classroom orchestration was much more difficult than in the Grade 10 classes where the organisation allows working with a half-class."

(MeTAH questionnaire answers, TE with Aplusix)

Unexpected events also occurred: the closure of the school due to students' sit-in (ETL teaching intervention with Cruislet), or the number of pupils' absences (Unisi teaching intervention with Aplusix).

Finally, some teams had to cope with the inadequacy of the school technological resources.

Synthesis

The following table synthesizes the elements of the global and of the local context which raised the greatest problems or concerns in the planning of the teaching interventions.

	Elements of the **global** context raising the greatest problems or concerns in **implementing**	Elements of the **local** context raising the greatest problems or concerns in **implementing**
Didirem Casyopée	Ø	Ø
Didirem Cruislet	Ø	School ICT Resources Student Population
ETL Cruislet	Ø	Unexpected event
ETL MaLT		Student Population
ETL Mopix	Ø	Student Population Schoolrooms
IoE MoPiX	Curriculum	Student Population School ICT Resources
IoE MaLT	Time	Student Population
ITD Alnuset	Ø	Ø
ITD Aplusix	Ø	Ø
MeTAH Aplusix	Curriculum	Schoolrooms
MeTAH Alnuset	Ø	Schoolrooms
Unisi Aplusix	Ø	Unexpected event
Unisi Casyopée		School ICT Resources

Table 4. Contextual issues and implementing of the teaching intervention

IV. Feedbacks on DDAs&PPs

One of the aims of the empirical research coordinated through the WP4 is to provide with feedbacks on both the DDAs functioning as didactical tools and the consistency of such functioning in relation to the theoretical assumptions, which guided the design of the DDA use in the classroom. In order to do that one has to carry out an analysis of the Pedagogical Plans (PPs) designed and implemented by the project partners, together with the analysis of the findings of the single Teaching Experiments (TEs).

With that respect there is the need of stressing once more that the planned experimentation is not aimed merely at investigating the didactical effectiveness of the designed PPs, and their impact on the school systems. Actually, particular attention is posed on the study of the DDAs exportability in different research contexts, and on the investigation of the possible different modes of use of the DDAs in an educational setting as well as of the hypotheses underlying these modes of use.

That reflects the importance given to the experience of the whole research cycle "design-development-user" of DDAs, where the term "user" does not refer only to students, or teachers, but also to math education researchers. The development of a cross-experimentation involving all DDAs was meant exactly to capture the complexity of the process across a range of situations and thus to contribute to feed the ITF in a rich way.

In this chapter, we synthesize and compare, DDA by DDA and PP by PP:

- the specific educational goals designed by the partners, and the partners' evaluation of the achievement of those goals,

- the modalities of use of the DDA according to the designed PP (that entails the consideration of more sub-dimension),

- the specific DDA features used,

- the educational hypotheses underlying the PP, and the evaluation made by the WP4 partners against the criteria defined a-priori and specified in the TE Portrait.

The synthesis we are going to present in the next sections is mainly based on the PPs produced before the implementation of the TEs, and on the TE synthesis and TE analysis produced by the partners after the implementation of the TEs.

The full specification of the educational goals, modalities of use, and educational hypotheses can be found partly in the ReMath PPs on-line depositary (http://remath.itd.cnr.it) where the PPs are stored and partly in the Annexes, where both the TEs analyses and the TEs syntheses are integrally reported.

Before comparing the different TEs, in the next section we will consider them separately and provide with an overview on how the partners evaluated their own TE with respect to both the achievement of the designed educational goal and the consistency of the theoretical hypothesis underpinning the PPs.

IV.1 Synthesis of the TEs

Looking at the single TEs, the first feedback, which one can receive from the experimentation, concerns trivially the didactical effectiveness of the teaching intervention (i.e. the implementation of the PP) and the consistency of the educational hypothesis underpinning its design. As previously argued, that is not our unique or main concern but it is

doubtlessly highly relevant to the objectives of WP4, which include to specification of new perspectives on a) means of using technologies to support learning, and b) investigating learning with technologies.

Before discussing more in details the results of the TE analysis, it is appropriate and necessary to remind that the analysis is still in progress: caution in drawing conclusions is advocated by most teams.

IV.1.1 On the achievement of the designed educational goals

A quick overview of the documents produced by the teams shows that in most TEs the educational aims designed a-priori were achieved (there are few exceptions which will be specified in the following). The table below tries to depict the situation, by distinguishing among the full achievement of the educational goals (FA), their (at least) partial achievement (PA), the not achievement of the educational goals (NA) and the impossibility (for now) to attest such achievement (IA).

	Aplusix	Alnuset	Casyopée	Cruislet	MaLT	MoPiX
Familiar	PA	FA	FA	FA	FA	PA
Alien	FA (ITD)	PA	PA	PA	PA	PA
	FA (Unisi)					

Table 5. Achievement of the educational goals

Though valuing this kind of synthetic presentation, one can recognize that it is not very much informative: many important hints escape this way of presentation.

First of all, one has to acknowledge that the evaluation of the level of achievement of the planned educational goals is at some extent relative to the "sensitivity" of the researcher, and may depend on different factors. One of these concerns the degree of integration of the designed Teaching Sequence in the reality of the classroom. To what extent did one succeed in taking into account the actual institutional or situational context of the teaching sequence implementation when designing the PP? How was the teaching sequence accepted in the school? in the classroom? by teachers? by students? How did it fit the curricular constraints? How did it fit the situational context (e.g. supposed students' prerequisites)? Did it fulfill teachers' and students' expectations? All that can make the difference. In addition different PPs and different DDAs may have different needs of integration: for instance some PPs are meant to be inserted in the school curricula as smoothly as possible, whereas other PPs are meant to bring immediate innovation into the curricula.

Hence besides the apparent uniformity emerging from the table 5, a certain complexity exists. For instance, when discussing the achievement of the educational goals, many teams seem to feel the need of re-stating their aims, clarifying them, adding remarks, or making distinctions.

Certainly, some teams can claim to have achieved the educational goals, exactly as they were a-priori formulated.

> "Our Educational Goal was expressed as follows:
>
> *To develop students' ability to recognise and analyse the properties of three dimensional geometrical shapes through construction and manipulation of 2D and 3D representations.*

Through the course of the teaching experiment, students were introduced to a range of ways of constructing representations of three dimensional objects and were successful in using these in structured and directed tasks."

(IoE analysis of the TE with MaLT)

Whereas, other teams, such as ITD and Unisi, propose a re-articulation of their original educational aims.

"The educational goals envisaged in our TE focus on the development of an operative and semantic control over algebraic expressions and propositions. In the following we describe our educational goals:

- Learning to practice the control of what variables and algebraic expressions indicate in an indeterminate way within a numeric domain using the quantitative method;

- Learning to practice the control of the relationship between two expressions using quantitative methods to distinguish among equivalent expressions, opposite expressions and reciprocal expressions;

- Learning to practice the control of the relationship between two expressions using formal methods to distinguish among equivalent expressions, opposite expressions and reciprocal expressions;

- Constructing a meaning for the notion of roots of polynomial and understanding the link between the roots and the polynomial factorisation;

- Constructing a meaning for the notions of equation, truth value of equation and truth set of an equation, equivalent equations, conditioned equality (an equality that is conditioned by the value assumed by the variable in the two expressions that are compared by means of the equal sign), identity.

The results of our TE have demonstrated that it is possible to achieve these didactical goals exploiting the mediation of Alnuset and in particular of the integrated use of the Algebraic Line and of the Algebraic manipulator."

(ITD analysis of the TE with Alnuset)

Or

"The general educational goals of the PP which has been specified in the portrait are:

1. Anticipating the introduction to the algebraic calculation, as a manipulation based on the equivalence.
2. Introducing to the *"structure sense"* of an expression.

Within such global aims, we pointed out more specific educational goals, that focus on numerical expressions in the perspective of introducing algebraic calculation. They are:

1'. acquiring the notion of equivalence between expressions;
2'. acquiring the structure sense for numerical expressions.

In particular, the role played by the properties of the operations to demonstrate the equivalence between expressions is considered a key point in the delicate passage from arithmetical to algebraic computations."

(Unisi synthesis of the TE with Aplusix)

More teams specify remarks, which were not included in the original plans. That is the case of ETL remarks concerning the educational goals of their TE with MaLT:

"Remark: Though all those aspects of the concept of angle in 3D space could be singularly pursued through the planned PP, it is not reasonable to think to be able of

pursuing all of them together. Actually, the choice of the specific educational goals to focus on, rests on factors like the didactic choices made by the teacher, the research focus of the researcher, the emergent perspectives during the implementation in the classroom, the progress of the activities and the available time."

<div align="right">(ETL analysis of the TE with MaLT)</div>

We highly value the emergence of such kind of modifications (re-statement, different articulation, remarks …). They do not mean that the PPs were not accurately designed, or that the PPs were not sound enough from the pedagogical point of view. Rather, these modifications attest the fact that PPs are "live" objects, under continuous refinement. That also confirms the suitability of the choice of the methodology of the TE for experimentation: the refinement of the PPs is made possible through taking into account the whole cycle – design, implementation and analysis. In fact the highlighted modifications result from the re-thinking of the designed PPs prior their implementation, from the implementation of the PPs themselves and from the analysis of such implementation.

There is also another important element, emerging from the analysis of the TE, which could lead to a refinement of the PPs. That is the specification or revision of the criteria for attesting the achievement of the designed educational goals. For instance:

"Thus, the achievement of the envisaged educational goal is attested through the analysis of students' verbal productions, in particular of the reports students were asked to produce at the end of each session.

Students can be said to have achieved the envisaged educational goals if:

a. they use specific terms (function; independent, dependent, geometrical, numerical… variable; graph; measure; domain; variation; co-variation; etc.) in "appropriate ways" (i.e. consistently with their (possible) mathematical meanings, the DDA functionalities and the specific activities at stake);

b. they relate mathematical meanings and processes to the software functionalities;

c. they express the main phases characterizing algebraic modelling of geometrical problems."

<div align="right">(Unisi synthesis of the TE with Casyopée)</div>

"The results presented above show clearly that Aplusix tasks posed much less, if any, difficulty, since the students worked only in the controlled mode and thus benefited from the feedback provided by the system that allowed them to succeed in solving the tasks proceeding by trial and error rather than analysing the structure of the algebraic expressions. However, as we can see, the success in solving conversion tasks with the system in controlled mode does not guarantee that the knowledge aimed at was used in the solving process."

<div align="right">(MeTAH synthesis of the TE with Aplusix)</div>

Feedbacks on the PPs also come from the teams' consciousness-raising of the difficulties met for implementing the PPs, and of their effects.

We tried to organize the introduction as foreseen in the Educational Hypothesis, but there were obstacles in using Cruislet in the institutional context: the French curriculum leaves little opportunities for not content oriented activities. The TPEs were foreseen as an opportunity, but it was actually difficult to persuade students to choose a project with Cruislet while keeping the TPE's spirit of free choice and open domain.

<div align="right">(Didirem analysis of the TE with Cruislet)</div>

IV.1.2 On the consistency of the educational hypotheses

Together with the achievement of the designed educational goals, WP4 partners agreed to investigate also the consistency between the possible achievement of the designed educational goals and the educational hypotheses underpinning the design of the PPs.

This decision reflects the shared conviction that attesting the achievement of the designed educational goals may be not enough for "validating" the PPs or for providing with feedbacks for refining the PPs. Consistently with the TE methodology adopted, investigating the consistency of the educational hypotheses is not only crucial for developing or refining theories on teaching and learning with ICT tools, but also for feeding the process of design-implementation-refinement of the PPs.

With that respect, teams were asked to *a priori* specify the criteria according to which one would be able to attest such consistency. When analysing their TEs, teams were asked to refer to those criteria.

Granted the unavoidable limits of a synthetic view, the table below is an attempt to give a snapshot of the kind of feedbacks on the educational hypotheses coming from the analysis of the TEs.

First of all, we distinguish between the TEs, whose analysis has progressed enough to provide with feedbacks on the educational hypotheses (F), and the TEs whose analysis has not progressed enough yet (no F). Then, as for the formers, we still distinguish between TEs giving "positive" (P) or "negative" (N) feedbacks on the hypotheses in focus. By "positive feedbacks" we mean feedbacks confirming (at some extent) the consistency of the educational hypotheses at stake or suggesting their possible refinement. By "negative feedbacks" we mean feedbacks seriously questioning the soundness of the hypotheses and asking for a profound revision.

	Aplusix	Alnuset	Casyopée	Cruislet	MaLT	MoPiX
Familiar		F-P	no F	F-P	F-P	F-P
Alien	F-P (ITD)		F-P	F-N	F-P	F-P
	F-P (Unisi)					

Table 6. Feedbacks on the educational hypotheses

The emerging picture is globally rather positive: most of the hypotheses assumed received a positive feedback, but the syncretic overall view risk to be still more affected and less informative than that provided in the previous section for synthesizing the class achievements.

First of all, hypotheses assumed by the teams are often "complex hypotheses", that is consist of a system of interrelated more focused hypotheses which concern and link among them different aspects of the teaching/learning processes in real class situations. The evaluation of this system of hypotheses cannot be captured under the label "confirmed" or "not-confirmed".

The following excerpt can show an example of the claimed complexity:

> "The possible confirmation of the hypotheses inspiring the design of the PP and linking
> the use of the DDA with students' achievement is questioned through the Re-CRQ.

In synthesis, answering the question whether an artefact functioned as a tool of semiotic mediation for some mathematical meaning requires to investigate different but certainly related aspects:

  a) the possible unfolding of the hypothesized *semiotic potential* of the artefact in relation to the designed tasks, and in relation to the target mathematical meanings;

[…]

  b) and the possible evolution of students' personal signs towards the desired mathematical signs and the possible development of a texture of different meanings related to the target mathematical meanings which (that texture) contributes to enrich already formed personal meanings.

[…]

  c) the possible exploitation by the teacher of the unfolded *semiotic potential* for fostering the evolution of students' signs towards the desired mathematical signs;

[…]"

(Unisi analysis of the TE with Casyopée)

This complexity brings and reflects on the kind and quantity of criteria against which the hypotheses should be confirmed, and consequently on the kind of data which may constitute evidence of the consistency of the hypotheses.

The following excerpt from IOE analysis of their TE with MaLT shows the difficulties to reconcile different hints coming from the analysis carried out against different criteria

"We proposed the following criteria:

  • *Students successfully use MaLT to draw 3D objects that fulfil the requirements posed by tasks set in the pedagogical plan or posed by the students themselves.*

All students experienced some degree of success in the tasks set but this was limited by the time available and their difficulties in learning to use the software.

  • *Students use the variation tool in MaLT and can explain its effect.*

Because of the context in which we were conducting the study (see below) we were able to make less use of the 2d variation tool than had been anticipated.

In Session 4, students used the variation tool in order to explore pre-constructed models (e.g. joining up prisms by dragging sliders representing angles and/or side lengths) and some were able to connect the values displayed on the variation tool with the movement of the shapes, in particular connecting the number 60 to the angle of an equilateral triangular face of a prism.

Other students, however, although they could successfully manipulate the sliders, had difficulty connecting the numerical display to the shape.

  • *In their presentations at the end of the project, students make use of different forms of representation for their virtual building that are consistent with one another.*

Each group of students worked to produce a poster displaying their design for the new sports centre and presented this to their colleagues in the final session. […]
The narratives given in their oral presentations to the class made some connections between the various components, but in general, the dimensions and shapes of different forms of representation were not consistent."

(IOE analysis of the TE with MaLT)

It is worth noticing that in order to deal with this underlined complexity, several teams decided to formulated explicit research questions addressing the consistency of the

educational hypotheses assumed: some of those teams investigated this issue in the frame of the CRQ (ETL TE with MaLT, ITD TE with Alnuset, Unisi TE with Casyopée) while other teams formulated specific research questions (ETL TE with Cruislet)

There is also another aspect which escapes the above synthesis. That is the reformulation of the educational hypotheses. In the previous section we pointed out that PPs are potentially in continuous refinement for their very nature, and we also pointed out that the TE methodology support the possibility of a continuous refinement. This feature regards also the possibility of reformulating hypotheses before, during or after the implementation.

The excerpt below reports ETL rationale for reformulating the educational hypotheses underpinning their TE with MoPiX.

> "The hypotheses as they were presented in the TE Guidelines (July 2007) indicate that the main educational goal (i.e. the students' construction of meanings about the role of the equations) was addressed only for a specific phase of the Pedagogical Plan, the phase in which the students changed the "Juggler" half-baked microworld. Having in mind that -in this phase- the students would engage in activities that were designed to be particularly challenging for them (i.e. play with the "Juggler" according to the existing rules, deconstruct the model underpinning its behaviour and modify it so as to express their personal ideas), we focused the wording of the hypotheses almost exclusively to this phase of the experimentations.
>
> However, the construction of meanings about the role of the equations was expected to also emerge during the first phase of the experimentations. For this phase, we had designed activities during which the students would be invited to deconstruct a model consisting of a single object (i.e. the "One red ball") so as to determine the object's behaviour and create a second object that would have the exact same behaviour as the first one.
>
> […]
>
> By reformulating the hypotheses made when designing the PP –not after receiving feedback from the experimentation and the analysis process, but after reconsidering the wording of the hypotheses as they were presented in the TE Guidelines (July 2007)- we try to make explicit how the use of the DDA is linked to envisaged educational goal. Each of the hypotheses incorporates in its wording the educational goal and thus confirmation of the hypotheses contributes to the attestation of the main educational goal."
>
> (ETL analysis of their TE with MoPiX)

We conclude this brief overview, reporting the result of Didirem Te with Cruislet:

> *"1. The framework TPEs makes possible*
>
>> *the use of Cruislet in the institutional context*
>>
>> *the actualization of the original potential it offers for working in 3D geometry*
>
> *[...].*
>
> *2. This requires careful introduction to this complex software through the choice and succession of appropriate tasks, a careful sharing of responsibilities between the students and the teacher, and a careful orchestration of the first phase of the instrumental genesis by the teacher.*
>
> We tried to organize the introduction as foreseen in the Educational Hypothesis, but there were obstacles in using Cruislet in the institutional context: the French curriculum leaves little opportunities for not content oriented activities. The TPEs were foreseen as an

> opportunity, but it was actually difficult to persuade students to choose a project with Cruislet while keeping the TPE's spirit of free choice and open domain. Thus expectations of Hypothesis 1 were not fulfilled. With regard to hypothesis 2, certainly time has been underestimated: most students were not comfortable enough with Cruislet as to carry out a project."

<div align="right">(Didirem analysis of their TE with Cruislet)</div>

Briefly speaking one may say that the educational hypotheses underpinning Didirem TE with Cruislet received a negative feedback from the TE enactment; the negative connotation of the feedback received is consistently reported in the introductory table. But if one looks more in details, (s)he can recognize that this label does not capture the essence of the situation and consequently does not give the TE its due. Actually one can notice that the educational hypotheses (in particular, the first one above) regarded contextual aspects and reflect the concerns expressed by the team since the very beginning of the experimentation:

> "We had concerns about the difficulty of bringing activities not directly consistent with the curriculum, in the mathematics course (scientific stream) at 11th grade where the syllabus to cover is very heavy."

<div align="right">(Didirem questionnaire on context for their TE with Cruislet)</div>

The unfortunate negative connotation of feedback on the original hypotheses brought to the formulation of new ones. The feedbacks coming from the implementation of this TE were reinvested in the design and implementation of a new TE whose still-on-going analysis cannot be unfortunately included in this deliverable.

IV.2 Cross analysis of the TEs

In the next sections we carry out a comparison, DDA by DDA, between the TEs designed and implemented by the different partners.

Keeping in mid the objective of contributing to feed the ITF, we will assume as a basis for the compared analysis the construct of Didactical Functionality upon which the ITF itself develops.

Hence, for each DDA, we are going to compare the related TEs, focusing on:

- Educational goals.

- DDA features.

- Modalities of use of the DDA. With that respect, a reduction is made: we chose to investigate (a) the possible design of familiarization activities for making students acquainted with the DDA, and (b) the tasks which the students were asked to accomplish through the use of the artefact.

The aim is to put into evidence the possible homogeneity or dis-homogeneity between familiar and alien TEs.

The analysis of most of the implemented TEs is still in progress; as a consequence this comparison cannot be exhaustive. More modestly the objective of the comparison is to identify paths for future possible analysis to be carried out in the last months of the project.

Alnuset

ITD (designer team) and MeTAH designed one TE centred on Alnuset each. Each TE was implemented in a class (grade 10 for both the implementation).

Educational Goals.

Each TE pursues a number of different related educational goals. Comparing those goals one can observe many differences: for instance ITD deal with (also) the notion of denotation of an algebraic expression, while MeTAH focused on the notion of function. Moreover, even if there is a common focus on the notions of equation and of equivalence of equations, the adopted perspectives appear well differentiate.

ITD: "The educational goals envisaged in our TE focus on the development of an operative and semantic control over algebraic expressions and propositions. In the following we describe our educational goals:

- Learning to practice the control of what variables and algebraic expressions indicate in an indeterminate way within a numeric domain using the quantitative method;
- Learning to practice the control of the relationship between two expressions using quantitative methods to distinguish among equivalent expressions, opposite expressions and reciprocal expressions;
- Learning to practice the control of the relationship between two expressions using formal methods to distinguish among equivalent expressions, opposite expressions and reciprocal expressions;
- Constructing a meaning for the notion of roots of polynomial and understanding the link between the roots and the polynomial factorisation;
- Constructing a meaning for the notions of equation, truth value of equation and truth set of an equation, equivalent equations, conditioned equality (an equality that is conditioned by the value assumed by the variable in the two expressions that are compared by means of the equal sign), identity."

MeTAH: "Let us remind that the main educational goals of the pedagogical scenario is that students construct:

1. the meaning of function as a relationship between dependent and independent variables;
2. the meaning of the notions of equation and inequation as statements that are true for some values of a variable;
3. the meaning of equivalence between expressions as statements that are true for all values of the variable;
4. the meaning of a solution of an equation as a value of the variable for which the equation is true."

DDA features

Both ITD and MeTAH exploited the potentialities of the Alnuset environment named "Algebraic line". This environment provides with a geometrical model (a couple of line) where one can represent algebraic expressions (letters, numbers, algebraic operations and algebraic expressions involving them). Letters are associated to free mobile points which can be dragged by the user. Algebraic expressions are associated to points which move according to the movement of the initial free points. Both teams exploited the fact that (i) each point

representing an algebraic expression is endowed with a label ('post-it') where the expression is specified and (ii) two equivalent algebraic expressions are represented through the same point and a label showing both the expressions.

In addition, ITD designed activities centred on the use of the 'Algebraic Manipulator' component (a structured symbolic calculator), whereas MeTAH envisaged to exploit the 'Cartesian Plane' component, where graph of functions can be displayed.

That difference is related clearly to the different educational goals pursued.

Modalities of Use: familiarization

ITD team did not plan an activity specifically devoted to familiarization. One could say that familiarization is step by step combined with pursuing specific educational goals.

At the beginning of most sessions the teacher is asked to gives some information about the commands of Alnuset which will especially used in the session.

Analogously MeTAH team combined familiarization with activities directly meant to pursue the designed educational goals. Only part of the first session is devoted to familiarization, but no specific familiarization activity is given. On the contrary, pupils are asked to explore the function $f(x)=x^2$, and the exploration of such function is directly related to the educational objectives designed.

Modalities of Use: tasks

As obvious, having different educational goals in mind, ITD and MeTAH teams elaborated different tasks. Anyway differences do not concern only the tasks designed, but also the designed uses of Alnuset, which in fact correspond to different principles inspiring the ITD and MeTAH PPs.

One of the principles underlying the design of ITD PP appears to be the assumption that learning develops through raising and overcoming contradictions. The designed use of Alnuset is intended to provoke and supply this process.

> "In this pedagogical plan, ALNUSET is used according to two different pedagogical strategies:
> - Task solution based on the use of pen & paper vs. task solution based on use of the tool
> - Explorative use of ALNUSET to find a solution to a task, followed by the question: What conclusion can you make?
> Both pedagogical strategies can be a source of contradictions, and overcoming these can be the motor of learning."
>
> (ITD PP with Alnuset)

The task have been consistently planned: each of them is considered susceptible of generating contradictions (i) between the solutions of different groups of students, or (ii) between the solutions in paper and pencil or within Alnuset of a same group of students. Hereafter some example of the tasks presented to pupils in different sessions:

> "Card1. EX: a) Write what a letter represents in algebra (for example the letter x). Is there any difference if the letter is considered in different numeric sets?

Insert the mobile point x on the algebraic line (AL) of ALNUSET. Move this point with the mouse and observe what x denotes in the following numeric set: Natural Integer, Relative Integer, Rational Numbers. Write some comments about the answer given above.

Card2. EX: c. Write two equivalent expressions and represent them in the AL. Use the AM to prove that they are equivalent. Write two non-equivalent expressions but equal for x =2. Use the AL to verify it.

Card4. EX: b) Consider the following expression: (x-2)/4. Try to imagine to represent this expression on the AL. Write a reciprocal expression under each movement of the variable x.
Verify your answer using the AL of Alnuset. Write what happen if you multiply the two expressions. Verify the answer using the AL of Alnuset. What conclusions can you carry out?

Card7. EX: a) Consider the following two polynomials: x^2+2; $2*x+3$.
Explain what putting the equal sign between the two expression means or in other words how you interpret the following writing $x^2+2=2*x+3$
What do you expect to find if you represent the two expressions x^2+2 and $2*x+3$ on the AL:
− the two points corresponding to the two expressions are coincident, whatever the value of x is.
− the two points corresponding the two expressions are coincident, only for some specific values of x
− other (specify)
Justify your answer
Represent the two polynomials on the AL. Drag the point x and verify your answer. Was you answer correct or wrong? Why?"

The idea of raising contradictions does not appear in MeTAH designed use of Alnuset. Instead, this DDA was conceived as an instrument for students' "guided exploration" of the properties of functions, equations and inequalities.

"Regarding these two functions, the students were first asked to observe the relationship of dependence between x and x^2 by noticing that, on the one hand, when x moves on the algebraic line, x^2 (or 1/x) moves accordingly and, on the other hand, x^2 (or 1/x) cannot by dragged with the mouse. Then, their attention was drawn to the way x^2 (or 1/x) moves when x moves on the line. The aim of this task was to develop an instrumental technique allowing to determine variations of a function. This technique is based on observation of the movement of f(x) when x is dragged along the algebraic line: when x and f(x) move in the same direction, the function f is increasing, when they move in opposite directions, f is decreasing."

(MeTAH synthesis of the TE with Alnuset)

Other tasks require pupils to solve equations or inequalities (without being introduced to Alnuset commands for solving equations and inequalities).

Aplusix

There have been three TEs centred on the use of Aplusix: a TE carried out by the team of Aplusix designers (MeTAH), another one carried out by ITD and another one carried out by Unisi. Globally four classes have been involved, grades 8 and 9, since 4 to 20 hours.

Educational Goals.

As for the designed educational goals we can notice a certain uniformity among the three TEs: in fact all of them meant to foster the students' development of the "structure sense" of numerical (ITD and Unisi) or algebraic expressions (MeTAH). Certainly there are some differences: MeTAH TE focuses also on procedural aspects of algebraic expressions, and Unisi TE focuses on the notion of equivalence between expressions.

MeTAH: "Help students to learn algebraic expressions (procedural and structural aspects) by articulating two different modes of representation for algebraic expressions: usual and tree representations."

ITD: "Understand the structure of numerical expressions."

UNISI: "[…] we pointed out more specific educational goals, that focus on numerical expressions in the perspective of introducing algebraic calculation. They are:

1'. acquiring the notion of equivalence between expressions;

2'. acquiring the structure sense for numerical expressions."

What is meant by "structural aspects", "structure" or "structure sense" is further elaborated by the teams, who provide with descriptions in terms of tasks that the students should be able to accomplish at the end of the teaching intervention.

MeTAH: "The students will be able to (curricular goals):

- identify the form of an algebraic expression given in either of the following representation systems: tree, natural language, symbolic language;

- convert an algebraic expression given in one representation system into another one;

- solve problems involving algebraic expressions given either in natural language or in symbolic language (usual representation)."

ITD: "In particular:

▪ Learn how to represent a numerical expression as a tree.

▪ Learn how to "build" a tree given a numerical expression or an expression described in natural language.

▪ Learn how to "read" an expression represented by a tree.

▪ Learn that there is only a linear expression for a tree while it could be different tree representations for an expression represented in linear form."

UNISI: "Following Hoch and Dreyfus (2006):

'A student is said to display structure sense for high school algebra if s/he can:

Recognise a familiar structure in its simplest form.

Deal with a compound term as a single entity, and through an appropriate substitution recognise a familiar structure in a more complex form.

Choose appropriate manipulations to make best use of a structure'."

DDA features

A certain uniformity, regarding the features of Aplusix considered, corresponds to the shown uniformity of educational goals. The attention is mainly posed on

- the possibility of representing expressions within Aplusix in two distinct ways: usual bi-dimensional representation, and tree-representation;

- Aplusix feedbacks concerning the equivalence of two "consecutive expressions". This feature is especially taken into account in Unisi TE, where this kind of feedback is directly related to the objective of fostering students' appropriation of the meaning of equivalence between expressions.

Anyway, as for the tree representation, one has to remark that MeTAH made students work alternately in controlled or free mode, ITD chose to use mainly the mixed mode, whereas the activity designed by Unisi asked students to work in free mode.
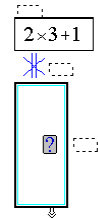
<u>Modalities of Use: familiarization</u>

In all the three TEs, specific introductory activities were designed and were explicitly labelled as "introduction to...". That does not mean that all the DDA features are introduced since the very beginning; actually some of them are introduced as the PP progresses.

A distinctive feature of those activities, common to the three TEs, concerns the designed roles of teachers and students: in fact the teacher has the responsibility of directly showing the functioning of Aplusix.

MeTAH: "In an ordinary classroom equipped by a computer with Aplusix-tree software and a videoprojector, the teacher manipulates the computer and manages class interactions."

ITD: "At the beginning the teacher gives some information about Aplusix tree representation. In particular he/she explains that there are two ways to construct a tree: the mixed representation and the controlled representation modes. The teacher explains that the focus of the lesson is to understand how constructing a tree from a linear representation of expression."

Unisi: "The teacher through an overhead projector introduces Aplusix to the students. The teacher shows:

- how to insert an empty line in order to carry out the solution of a task;

- how to write fractions in SR."

In addition, for those introductory activities, Unisi made explicit specific instrumental goals, whereas did not mention any curricular goals.

Unisi: Introduction to the software and in particular to SR. […] Interpretation of the feedbacks of Aplusix. […] Building a tree; understanding of the feedback given by the software.

ITD: Card 1: to introduce students to the tree representation of a numerical expression making use of two specific functionalities of Aplusix: the mixed tree representation and the controlled tree representation.
EX: 2) Write the expression 2*3+1.
By the mouse right button select "Step_New Step". An empty cell appears.
Construct the tree after selecting "Mixed_Representation".
You can expand the representation using the button +.

Finally we observe that different time-slots are allocated for introductory activities: about 1 hour for MeTAH and ITD TEs, and more than 2 hours for Unisi TE.

<u>Modalities of Use: tasks</u>

Generally speaking the kind of tasks proposed to students are very similar in the three TEs. In fact the tasks are mainly tasks of conversion between 3 different systems of representation of numerical or algebraic expressions: natural language, standard bi-dimensional representation, and tree graph representation.

Besides the similarity, there are important differences due to the mode of use of Aplusix tree-representation allowed in the TEs: free mode, controlled mode, mixed mode.

In addition the use of Aplusix is different in relation to the task: ITD made students use Aplusix mainly as a "validator", that is students were asked to solve given tasks in paper and pencil and then to verify the correctness of their answers using Aplusix. On the contrary MeTAH and Unisi required students to solve the given tasks directly using Aplusix (whenever it was possible).

Hereafter there are some examples of the tasks proposed to students:

MeTAH: "(ES1.2) Conversion between natural language (RNL) and tree registers (RT):

RNL -> RT with Aplusix in controlled mode

RNL -> RT with Aplusix in free mode

RT -> RNL in paper and pencil"

ITD: "1) In the right space of the table write the linear expression of each tree representations

2) Verify if each linear expression, written in the table, corresponds to the tree representation, writing in Aplusix each linear expression and transforming it in a tree. In Aplusix use the feedback provided by the system to see, step by step, if your construction is correct. Is the tree representation in Aplusix the same tree representation that you find in the table?"

Unisi: "Open the file Esercise1.alg in Aplusix. You will find on the screen the image reported here after:
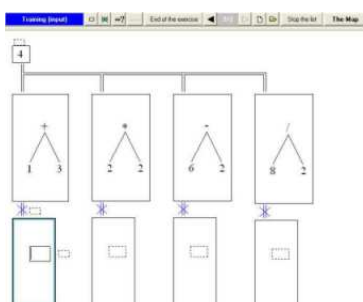
The number 4 is expressed in four different ways by means of **tree graphs** (in short, trees). Each tree is made of a **node** and two **branches**:

Continue filling in, maintaining the same operator on each column (operator + along the first column, operator * along the second column and so on). Build at least two trees for each operator.

Try to invert the **leaves** of the trees (namely the two operands). What can you observe?

Try to express in **standard representation** the trees provided and those that you have produced in the file Esercise1.alg."

To conclude one can remark that:

1. the given tasks are not "usual school tasks": they are not part of the usual national curricula of any of the Countries where the TEs were implemented, even if the TEs were meant to pursue objectives pertinent to the respective national curricula;

2. the given tasks have a clear mathematical sense which is independent on the DDA, even if, certainly, posing them within the DDA or not makes the difference.

Casyopée

There have been two TEs centred on the use of Casyopée, respectively designed by Didirem (DDA designers) and Unisi. Six classes (grade 11 and 12) participated to the experimentation; the enactment of the PP required 8 to 13 hours.

<u>Educational aims</u>

As far as educational goals are concerned, Didirem and Unisi PPs share important similarities. Both the PPs aim at fostering students' appropriation or consolidation of meanings related to the notion of function (function as co-variation, variables, domain of a variable) and of meanings related to modelling process in algebraic settings.

DIDIREM: "The pedagogical plan aimed to help students to construct or enrich knowledge on two
>   aspects:
>   1. meaning of functions as algebraic objects,
>   2. meaning of functions as means to model a co-variation in geometric and algebraic settings.
>   More specifically:
>   as for the notion of function as an algebraic object, students should consolidate:
>   >   the meaning of variable,
>   >   the distinction between variable and parameter,
>   >   the meaning of function of one variable with several registers of semiotic representation,
>   >   the fact that a same function may have several algebraic expressions,
>   as for functions as means to model a co variation, students should develop:
>   >   the ability to experiment and anticipate in a dynamic geometric situation."

UNISI: "The main goals envisaged when designing this Pedagogical Plan are to foster the evolution of
>   students' personal meanings towards:
>   (a) the mathematical meaning of function as co-variation and thus consolidate (or enrich) the meanings of function they have already appropriated;
>   (b) the mathematical meanings related to the processes characterizing the algebraic modelling of geometrical situation.
>   More specifically,
>   >   as for the notion of function, students should consolidate or enrich:
>   - the meaning of variables both geometrical and numerical,
>   - the meaning of domain of a variable,
>   - the meaning of function as co-variation over time (even when different kinds of variables are involved),
>   - competencies related to the passage between different representations of function (at least, algebraic and graphical ones)
>   >   as for the modelling process, students should learn to:
>   - recognize geometrical variables,
>   - associate numbers (numerical variables) to geometrical variables,
>   - associate geometrical variables to numbers (numerical variables),
>   - pass from not-measurable geometrical objects (e.g. points) to measurable geometrical objects,
>   - parameterise (optimize the number of variables),
>   - express the relation between numerical variables through formulas."

DDA features

One can notice certain uniformity also regarding the DDA features in focus.

Both Didirem and Unisi try to exploit the potentialities of the Dynamic Geometry environment, and of the Geometric Calculation sub-environment. Besides the usual features of dynamic geometry environments, the main common features at stake concern the possibility of defining, in the geometrical settings, functions whose independent and dependent variables can be constructed (through the commands "create function", "choose variable" and "create calculation" respectively) starting from geometrical objects in the Dynamic Geometry environment. The function created in the geometrical environment can be exported in the algebraic window. Another important feature is the possibility of defining geometrical objects depending on objects defined in the window (e.g. parameters).

As for the algebraic window, the main characteristics, exploited in both the TEs, concern: (a) the possibility of displaying graphs of functions and exploring those graphs, and (b) the possibility of defining parameters which can be "exported" in the geometrical environment.

In addition, Didirem PP includes activities were students are asked or to use usual features of CAS, which are implemented in the algebraic window.

Modalities of use: familiarization.

Unisi devotes to familiarization the first session (2 hours). No presentation by the teacher is planned; instead students are given a set of tasks aiming at exploring the functionalities of the DDA and obtaining an overview of Casyopée features. The given tasks aim at guiding students to observe and reflect upon the "effects" of their interaction with the tool itself. A*d hoc* tasks are designed for that purpose, e.g.:

> "Could you choose a variable acceptable by Casyopée and click on the 'validate' button? Describe how the window 'Geometric Calculation' change did after clicking on the button. Which new button appeared?"

> (Unisi Casyopée-centred PP)

Familiarization as well as the whole pedagogical plan are not meant to make students able to use Casyopée for accomplishing given tasks, but to foster the students' consciousness of the mathematical meanings at stake.

On the opposite, Didirem team pays specific attention to a progressive introduction to the DDA combining the use of the artefact and the students' mathematical knowledge: students work only in the algebraic window during section 1, then only in the geometrical windows in section2; finally section 3 gives an opportunity to reinvests and combine what already learnt in the previous sections about the two separate environments. Moreover, all the tasks proposed are mathematical ones and highly consistent with scholastic tasks. All the tasks are designed in order to permit students to progress autonomously. Students face the problem and are expected to construct new knowledge. Great value is given to the interaction with the DDA, based on the possibility of feedbacks. When needed, the teachers opens the activities demonstrating the specific features of Casyopée at stake.

Because familiarization with the use of Casyopée is step by step combined with pursuing specific educational goals, it is quite difficult to isolate familiarization activities from other activities. Actually in the case of Didirem TE with Casyopée thinking in terms of familiarization is inappropriate.

Anyway, in order to develop some kind of comparison, one might pragmatically consider those activities in which the instrumental goals are preponderant with respect to the curricular ones.

An example of the tasks used in such activities is the following:

Students are provided with the graph of a quadratic function, h, whose algebraic expression is hidden. After creating in Casyopée two parameters (*a* and *k*), and the functions respectively defined by f(x)= x² and g(x)= f(x+k) + a, the students are asked to find the values of a and k such that the graph of g coincides with the initially given graph.

Knowledge related to the "associated functions" is revisited in this task. Using the tools provided by Casyopée students are expected to consolidate what they already know but also are expected to deepen their understanding of the functioning of parameters in respect to the graph.

Modalities of use: tasks

Besides familiarization, Unisi and Didirem planned to propose students usual optimization problems formulated in a (Euclidean or Cartesian) geometrical setting.

Such kind of tasks is perfectly consistent with the respective national curricula, and could be formulated without any reference to Casyopée.

As we are going to show, the two problems chosen by the two teams for their PP share the same mathematical core but present differences in their didactic adaptation.

As for the sequence of activities designed by Unisi, they start with presenting to the students a "complex" optimization problem, posed in a geometrical setting and formulated in generic terms, e.g.:

> "Given a triangle, what is the maximum value of the area of a rectangle inscribed in the triangle? Find a rectangle whose area has the maximum possible value."

The aim is to elaborate on those problems so to reveal and unravel the complexity and put into evidence step by step the specific mathematical meanings at stake. According to the designed pedagogical plan, the teacher plays the delicate role of guiding students to unravel such complexity and to make the targeted mathematical meanings emerge.

On the opposite, the sequence of activities designed by Didirem starts with a particular case and concludes proposing the 'same' problem in more general terms. This last problem aims to synthesize many of the aspects emerged in the previous activities, and consequently require students to re-invest what they achieved with respect to both competencies with Casyopée and mathematical knowledge acquired.

> **"Problem.** a, b and c positive parameters. A (-a, 0) et B (0, b) et C(c, 0). Find a rectangle (MNPQ) with M on [OA], Q on [OC], N on [AB] and P on [BC] with a maximal area?"

Cruislet

The TEs centred on the use of Cruislet were designed and implemented by ETL team (designer) and Didirem team (not-designer). Globally five classes have been involved so far, of different grades (10 and 11), since 5 to 28 hours.

Educational Goals

ETL and Didirem educational goals appear profoundly different one from the other. Even if they only share a common reference to the notion of vector, such references appear still vague (as for Didirem TE) or feeble (as for ETL TE). As a consequence, one does not expect to be able to establish clear connections between the students' achievement in the two TEs.

ETL: The educational goals specified in the PP, concerned mainly the concept of function as well as the notion of vector as the displacement in both geographical and spherical coordinate systems. Specifically, the educational goals included:

- The exploration of the concept of function as co-variation using the geographical coordinates as a system of reference

- The development of the notions of dependency between airplanes' positions.

- The study of the existence of a rate of change of relative displacements on the 3d space.

- The exploration of the notion of the vector as the displacement in Cartesian and in polar coordinate system

- The study the notions of geographical coordinates as the variables of the vector of displacement in Cartesian and/or in polar coordinate system.

[…] Regarding the concept of vector, we should mention that our initial aim was not for students to study this concept in depth, but rather to use it as a vehicle to displace the airplanes through the use of the corresponding systems of reference. Thus, vector became an object with which students engaged in navigational activities and through this activity, they developed mathematical meanings.

DIDIREM: Use the potential offered by the DDA for having students working on 3D realistic problems and enriching the meaning they give to vectors through the use of representations non standard at high school level and the meaning they give to curves such as circles, spirals or helix through a local generation of these.

DDA features

As for the DDA features, the teams do not make clear whether they pose attention to specific characteristics or representations provided by Cruislet.

ETL focus globally on (1) the presence of the Logo editor, and on (2) the features of the "terrain scene" including in particular the possibility of working with two systems of coordinates: the geographic and the spherical ones.

Didirem pose their attention on the possible ways of navigating: making use of the mouse or running Logo procedures to pilot avatar.

Apart from the evident differences, both the TE were meant to exploit Cruislet affordances for multi-representations.

Modalities of Use: familiarization

Both the TEs contemplate a familiarization session for introducing pupils to Cruislet functionalities at stake. In both of them, the teacher presents Cruislet to the class: particular (s)he presents (i)how to create an avatar, (ii) the different existing modes for moving this avatar, (iii) the camera system, and (iv) some functionalities of the Logo editor.

Immediately after in the session, the students are asked to:

ETL: "experiment with:

- position properties (Lat, Long, Height) that define the position of the airplane and are related to geographical coordinates. […]

- geo-coded information provided by the environment in the Content Tab (e.g. in order to define specific coordinates such as the airport of Athens). […]

- camera properties: in order to find out where the airplane is placed."

DIDIREM: "participate to the collective programming of a first trip with one escale, for instance a flight from Athens to Samos, with a stop in Mikonos or a circular flight (Athens, Iraklion, Rhodes, Athens). And then they are asked to modify the collectively designed trip."

Modalities of Use: tasks

ETL and DIDIREM teams elaborated very different tasks.

First of all, in ETL PP one cannot find specific tasks.

ETL makes explicit the kind of task which can be proposed to students, as well as the rationale of that kind of tasks. More specifically such tasks are centred on the relationship between the displacements of two avatars: students are asked (i) to guess the (hidden) relationship between the displacements of two airplanes through exploring their behaviour displayed in Cruislet environment, (ii) to construct a relationship between the displacements of two airplanes, and (iii) to guess the (hidden) relationship constructed by their schoolmates. But the exact formulation of the tasks is missing. Apparently the teacher is given the freedom and the responsibility of the choice basing on ETL hints:

> "The activity could be conceived as a game as students move the airplanes having a particular goal. The object of the game is to land the second airplane at Thessaloniki, where the second airplane (the spy) disappear and the first is getting free from the espial. In order to do this they must find the dependence relationship and move the first airplane in a position where the second airplane would be at Thessaloniki."

> (ETL Cruislet-centred PP)

DIDIREM tasks instead were centred on modelling and simulating. In fact students are asked to use Cruislet for planning flights over Greece, for instance:

> "Someone has prepared a flight from Athens to Heraklion. The altitude of cruise is 2000m. Program the flight and its visualisation. Unfortunately the programmer has not taken into account the wind. The wind comes from North-West and its force is 40km/h. The cruise speed of the plane is 200km/h. What is the real trajectory of the plane?

> A variant of this problem: a pilot has prepared a flight from Athens to Heraklion airport. He has planned that after 45mn of flight he would be just above the centre of Milos Island At the estimated time, he is above a small island, north east of Milos. How to correct the trajectory? Visualise the flight without and with correction."

Comparing these kinds of tasks one can highlight that they share a common general aspect and that an equally general aspect distinguish them, infact:

- neither of them are mathematical tasks *stricto sensu* - the formers are game-like tasks intended to challenge students, whereas the latters are presented as "real life problems" - even if they certainly can engage students in some kind of mathematical activity;

- ETL tasks live within the DDA, they are inescapable linked to the DDA features and hence they cannot be exported. Cruislet is not merely a tool for accomplishing those tasks, more appropriately it gives sense to them. Instead, the task proposed by Didirem is independent on Cruislet; it is the task of programming a flight, of re-arranging it, etc. Certainly though independent on Cruislet, the tasks were designed having in mind their realization in Cruislet.

MaLT

MaLT DDA is designed and developed by ETL, and it has been experimented by ETL themselves and IoE. The experimentation involved two classes, grade 7 and 8, for 18 and 8 hours respectively.

<u>Educational aims</u>

MaLT was designed with the general objective of fostering the development of students' mathematical meanings related to 3D geometrical notions. The educational goals of ETL PP are more focused and concern the development of student's mathematical meanings for the concept of angle in 3D space.

> "The [above] main educational goal have been articulated in the following specific educational goals in the MaLT PP:
>
> - exploring the notion of angle as turn and measure within the 3D space (e.g. the notion of angle as a change of direction and planes in 3D space, the notion of angle between two different planes, the notion of angle between two different 3D figures)
>
> - identifying the mathematical structure of 3D geometrical figures (e.g. distinguish the different 2D planes of the construction and relate it to the type/number of angles)
>
> - identifying the geometrical properties of 3D objects (logical arguments to justify conclusions, relationships among angle, side lengths, perimeter areas, and volume, develop intuitions and conjectures about the geometrical properties and relations of parallelepipeds)
>
> - understanding the relation between 2D and 3D representations when using the former to construct simulations of real 3D objects (e.g. identify the role of the 'repeat' command concerning 2D shapes in the construction of a 3D geometrical figure - between rectangle and revolving doors)
>
> - developing fluency with the mathematical expressions to describe a 3D geometrical construction with variables
>
> - identifying the role of variables in the construction/manipulation of 3D geometrical figures in different sizes."

<p align="right">(ETL analysis of the TE with MaLT)</p>

When defining their educational goals, IoE assumes a wide perspective fully consistent with the general scope of the design of MaLT: let students make meanings in relation to 3D geometry. It is worthwhile noticing the importance explicitly given to students' semiotic activity related to the use of the DDA, exactly when defining the educational goals of the PP.

> "In particular we are interested in
>
> - The different modes of communication students make use of when interacting with the 3D geometrical shapes,

- The choices students make between and within semiotic systems (modes) in order to communicate their completed design to their peers.

- The ways in which the properties of shapes are represented in different semiotic systems."

<div align="right">(IoE synthesis of the TE with MaLT)</div>

DDA features

In their PP, ETL planned to exploit several features of MaLT:

- Turtle Scene, a 3d grid-like interface in which a 3d turtle is visualised.

- The Logo-like programming interface, linked to the Turtle Scene, and enriched with new Logo commands.

- The Variation tools, which are a set of tools for dynamically managing the variation of the values of the variables defined in the Logo procedures. It consists of three kinds of variation tools, the Uni-dimensional Variation Tool, the Two-dimensional Variation Tool and the Vector Variation Tool.

IoE make use of the many of those features, in particular they limit the use of the Variation tools to uni-dimensional and two-dimensional variation tools.

Modalities of use: familiarization

As far as familiarization is concerned, one can remark that according to ETL PP students are supposed to have already some familiarity with the basic 2d Logo commands. Provided that, familiarization (2 teaching sessions) with MaLT is made through involving students in two specific tasks: make the turtle move so to simulate the taking off of an airplane and make the turtle move in a way that its trajectory (displayed in the Scene) can evoke the windows of two consecutive walls of a room.

The former is thought to familiarise students with moves and turns in the 3d geometrical space of MaLT. The latter is meant to foster students' production of conjectures concerning angular relationships.

ETL PP is not mandatory, and the teacher is given a certain freedom in managing these sessions, nevertheless no formal presentation of the DDA is foreseen.

On the contrary, when designing its PP IoE did not assume that students had previous experience with Logo programming. Students were supposed to be introduced to the Logo commands and syntax necessary to move in 3D space within the PP. The introductory tasks regarded the development of Logo programming for "producing" parallelograms and parallelepipeds through the turtle movement.

Actually, the difficulties of introducing Logo programming were underestimated:

> "the time available was insufficient and the students had low confidence and, in many cases, poor motivation, and were unused to collaborating with one another. Their lack of previous experience with Logo meant that progress with MaLT was slower than anticipated."

<div align="right">(IoE synthesis of TE with MaLT)</div>

Modalities of use: tasks

In ETL PP there are two main global tasks: constructing within MaLT the "simulation" of a revolving doors and constructing within MaLT the "simulation" of a spiral staircase. Each of the two tasks is planned to engage students for several teaching sessions (4 and 2 sessions respectively). During those sessions, the main tasks are articulated in "sub-tasks", respectively:

- Simulating the opening and closing of a door.

- Constructing a revolving door simulation.

- Extension: Simulating the opening-closing pages of a book.

And

- Constructing a stair.

- Constructing a spiral staircase simulation.

In addition the teacher is expected to stimulate the students through prompting them, giving hints, posing questions and bringing into the foreground points for consideration.

The leading idea of the activity designed by IoE is to present students with a whole "complex" project to be carried out: to design a new sports centre for the school. Carrying out such a project entails several activities (inside and outside the classroom): to identify the space within which it is to be built, to draw plans to scale the space available, and so on. Within this frame, MaLT is introduced as a tool for designing specific components of the buildings: doors and staircase.

Concluding, the tasks used in both the PPs are not strictly mathematical tasks, and cannot be exported outside MaLT, or better, being tasks of simulation they make sense only with respect to some tool supporting that kind of simulations.

MoPiX

MoPiX was experimented by IoE (the designer team) and ETL (the experimenter team). The realization of the TEs involved one class each, respectively for 15 and 25 hours. Students were 16 to 21 years old.

Educational Goals

Both IoE and ETL pay attention to aspects related to motion in their TEs, but such common reference is declined in different forms in the two TEs. That diversity is already evident when considering the educational goals. In fact the development of specific concepts related to motion is an explicit educational aim for IoE TE, while ETL TE is oriented towards the development of meanings related to the equations describing phenomena such as motion. More precisely, ETL do not mean to *a-priori* link educational goals to specific mathematical notions.

IOE: "The envisaged educational goal of the teaching experiment was the development of students' concepts of motion in accordance with Newtonian laws. In the implemented pedagogical plan, this focused primarily on the development of concepts of velocity and acceleration. Specifically:

- velocity as change in displacement,

- velocity (in a plane) as a two dimensional vector, either (magnitude, direction) or (horizontal magnitude, vertical magnitude) - the second of these being most naturally encoded in MoPiX notation,

- velocity remains constant unless acted upon,

- acceleration as change in velocity,

- acceleration as a force - specifically acceleration applied at an instant."

ETL: "The main educational goal addressed through the design of the MoPiX Pedagogical Plan […] concerned the students' construction of mathematical meanings regarding the role of the MoPiX algebraic equations and the relationships between them, while representing phenomena such as collisions and motions and experimenting (e.g. constructing, deconstructing and reconstructing) with the corresponding animated models. […] Thus, the main educational goal addressed a the Pedagogical Plan was deliberately not correlated to a specific mathematical concept or domain, as it would be for example the concept of function or the notion of the variable.

DDA features

Globally the same features of MoPiX have been taken into account by IoE and ETL teams when designing their TE:

- The existence of a set of pre-defined equations, from which users may select those to allocate to an object [Equation Library].

- The possibility of allocating equations to the object whose behaviour and property are defined by those equations, once the play button is pressed [Dragging and dropping equations].

- The possibility of revealing the set of equations defining the properties and behaviours of an object [Flipping objects].

- The existence of equations linking among them the behaviours and properties of different objects [Multi-objects equations].

- The existence of an Equation Editor which may be used to create new or to edit existing ones [Equation Editor].

Modalities of Use: familiarization

According to the original plan IoE devoted the first two-three sessions to introduce students to MoPiX (4-5 hours)

"The activities proposed in this section are intended to introduce students to the basic functionalities of the tool and to the fundamental idea that equations are used to describe and control the behaviour of objects.

There are two main parts to this section:

1. use of equations to define and control objects

2. graphing"

(IoE MoPiX-centred PP)

A detailed description of the familiarization phase and of its rationale is provided in the PP.

The teacher is asked to demonstrate the kind of animation that can be produced in MoPiX by loading and running an already built model from the web-based library, and to demonstrate the functioning of some of some of the features previously mentioned: the flipping feature, the equation library, the dragging and dropping feature. The teacher also shows and discusses the effects of allocating equations to an object. Students are initially asked to follow the same procedures to reproduce the same effects, and then they are asked to freely explore the effects of the equations available in the initial library. The teachers should stimulate students to explore in particular the equations of motion.

ETL devoted four sessions (10 hours) to familiarization through a sequence of activities named the"One-Object Equations" sequence. This sequence of activities introduces students to the MoPiX environment and aims at giving them the opportunity to achieve a certain level of familiarity with the environment and its functionalities as they interpret, manipulate and edit equations in order to control existing and create new animated models. The familiarization with the MoPiX environment does not constitute a discrete phase where the teacher gives out a fixed set of instructions for the students to follow or a process during which the teacher demonstrates the main functionalities of the environment and the students repeat the teacher's actions step-by-step, following the exact same procedure. The "One-Object Equations" provides a context that allows students to explore the computational environment by themselves and at the same time permits them to engage in activities personally meaningful to them.

The activities of this phase aim at engaging students to the deconstruction of an already built model and the construction of new ones. During this phase students are asked to load and execute a pre-defined model, to observe the generated animation, and to figure out what determines the behaviour of the displayed object (the simulation of a bouncing ball). They are then asked to reproduce the ball's motion using the equations themselves and near the end of this phase the students are encouraged to express and implement their ideas, constructing new models. At each time, students are asked to interact with DDA and among them, to share their ideas, to formulate hypotheses, to develop strategies to test these hypotheses and engage in joint decision-making processes, draw conclusion, negotiate and argue about the validity of their conclusions. Hypotheses and strategies are then shared and collectively discussed. In this context the role of the teacher is mainly to animate the possible discussions among peers.

> "the teacher doesn't intervene by giving out information concerning the 'correct answer' or by indicating a certain procedure to follow […]
>
> The teacher could act as a facilitator and co-ordinator, encouraging the members of each workgroup to question both their own ideas and the ideas presented by others, to reflect on their current understating and refine the models they have created."
>
> (ETL MoPiX-centred PP)

Modalities of Use: tasks

IoE built their PP around two kinds of tasks: the so-called "debugging task" and "challenges".

After the familiarization phase, students are faced to a task of debugging, that is they are presented with a pre-built (buggy) model supposed to simulate the trajectory of a projectile. Students are asked to examine the model, conjecturing the reason for its "malfunctioning", adding, removing or changing existing equations so to obtain a correctly working model.

The challenges are open tasks requiring students to produce models for obtaining given effects. There are three challenges proposed in the PP:

1. creating a model of a bouncing ball;

2. simulating throwing a ball;

3. collaborating with other students to produce a complex animation with several interacting components.

These are very open tasks, students can cope with them in many different ways and there is a wide variety of possible models which students can produce to get the required effects. In addition these challenges can be further varied and extended by students.

> "Students are challenged to construct a bouncing ball. This may be extended to create a simulation of a ball thrown against a wall or to explore the effects of different gravitational forces."

> (IoE MoPiX-centred PP)

After the first phase, students are introduced to a pre-built model involving multi-objects equations, the so-called "Juggler", consisting of three interrelated objects: a red ball and two rackets with which the ball interacted.. In the remaining sessions (15 hours) the class activity is organized around this model. Students are engaged in the following activities:

(a) deconstructing the already existing multi-object equations model

(b) modifying the already existing multi-object equations model and and using it as a starting point to construct new ones

(c) sharing the newly developed models.

The formulation of the tasks students have to accomplish is generic and really open.

> "In this phase the teacher asks the students to perceive themselves not as the kind of persons who play with the "Juggler" microworld without being able to modify anything in the environment (simple users) but primarily as the kind of persons who have deep access to the underlying model of the game and thus have the right to define the behaviours and the properties of the objects they choose to comprise in the environment (designers and programmers)."

> (ETL MoPiX-centred PP)

That allows students to possibly engage in different kinds of activities:

> "The students will decide for themselves about the properties and the behaviours assigned to the objects as standalone objects or with regard to others objects present or inserted on the "Stage". The activities students could engage could be divided in different categories:
>
> (a) Activities aiming at changing the way the objects move
>
> (b) Activities aiming at changing the way the objects connect to each other
>
> (c) Activities aiming at inserting new objects on the "Stage" and assigning them behaviours and properties."

> (ETL MoPiX-centred PP)

Concluding, with respect to the kind of tasks designed, one can attest certain uniformity between the perspectives assumed by IoE and ETL in their respective PPs: both of them propose to students tasks which cannot be exported outside the DDA and which are not "standard mathematical tasks".

V. Analysis of the answers to the Re-CRQs

The long discussion carried out among the teams on the design of the cross experimentation led us to recognize the prominent role of re-formulating clear research questions: not only for their function in leading the local experimental designs, which are intended to provide answers to such questions, but mainly for their potential capacity of linking the different experimentations. That led to the conclusion that *sharing research questions* would have been a potentially effective tool to limit fragmentation and foster the possibilities of comparing and integrating the single Teaching Experiments results so as to allow producing common results common results enhancing our understanding of meaning-making through representing with digital media.

In the first chapter we outlined the process carried out to develop a methodological tool to address the issue of producing common results. Such tool was centred in the formulation of the Minimal Theoretical Framework and the related Common Research Question, which we remind for the reader's convenience:

> *How can the representations identifiable in the DDAs be put in relationship with the achievement of specific educational goals?*

All the experimenting teams reformulated the CRQ in their own theoretical terms, preserving the essence of the CRQ itself, and addressed it through their local Teaching Experiments. They also specified which *top research concerns* are related to their reformulation of the Common Research Question. Those Reformulations of the CRQ and the related concerns are specified in the TE Profiles.

The analysis of the TE should provide answers to the corresponding Re-CRQ. It was agreed that such answers should be expressed in the terms and according to the language provided by the Theoretical Framework which each team adopted. In order to facilitate comparisons among the results, teams were asked to:

    i.   articulate the answers making reference to our developed common language, namely: the construct of DF, the MTF and the language of concerns;

    ii. try to distinguish as far as possible between observation and elements of observation and their interpretation;

    iii. specify which concerns drove the analysis of the TE itself and how.

Now we have to face the task of comparing the specific answers to the Re-CRQs get from the respective TEs. The process of meaning making through digital media is addressed in our CRQ through investigating three key-dimensions:

- The educational goals envisaged
- The representations identified in the DDA
- The relationship between those representations and the achievement of the envisaged educational goals

We will start our analysis investigating how those dimensions are considered in each TE.

The table below is meant to provide an overview of how those dimensions appear in the formulation of the Re-CRQ. In the next sections we will analyse more in details how those dimensions are considered in the Re-CRQs and the answers provided.

	Representations identified DDA features	Relationship	Educational goals
Did Cas	Representations of geometric functions. Geometrical computation; computation of dependence (feedbacks linked to variables choices); representations of parameters and variables.	The use of the DDA affects students' understanding the representations provided by the DDA affect the understanding (or the competencies)	understanding of functions understanding of the idea of functional dependence in geometrical situations; competency to choose adequate variables in specific geometrical situations; parameters and variables
Did Cr	*CRQ for the first experiment (11th grade):* Cruislet representation of 3D displacements *CRQ for the second experiment (9th grade):* semiotic system offered by Cruislet	*CRQ for the first experiment (11th grade):* Be a basis for *CRQ for the second experiment (9th grade):* tasks allow students to make sense of the complex semiotic system and use for solving tasks involving…	*CRQ for the first experiment (11th grade):* develop project teamwork *CRQ for the second experiment (9th grade):* 3D coordinates, vectors, displacements and curves
ETL Cr	spherical and geographical systems of reference in Cruislet	Using spherical and geographical systems of reference, students construct meanings	construct meanings about the concept of function
ETL Ma	representations in MaLT	Student use representations to construct meanings	construct meanings for the concept of angle in 3d space
ETL Mo	representations in MoPiX	students construct mathematical meanings while using the representations in the DDA to construct virtual models	mathematical meanings about the role of the equations
IoE Mo	MoPiX the variables x, y, Vx, Vy, Ax and Ay the semiotic systems offered by MoPiX	concepts are represented through students' semiotic activity in the context of use of the DDA students operate with the variables x and y, Vx and Vy, Ax and Ay students use forms of language to communicate about	concepts of motion velocity and acceleration
IoE Ma	MachineLab variation tool	students make meanings through their semiotic activity in the context of working with	Meanings in relation to three dimensional geometry

	semiotic systems (in general?)	students make interpretation of the effect of using students communicate	
ITD Al	algebraic representations mediated by Alnuset	educational practices mediated by DDA are useful to construct meanings for	meanings for the use of letters in algebra and to understand what algebraic expressions denote meaning for equations
ITD Ap	different representations (linear representation, tree representation, natural language)	educational activity mediate the understanding conversion effective to teach how…	understanding of the structure of numerical expressions represent numerical expression as a tree build a tree corresponding to an expression in natural language read an expression represented by a tree
MeT Ap	Semiotic register "tree representation" and its articulation with the usual and natural language registers	semiotic register and its articulation help the students to understand	understand the structure of algebraic expressions
Met Al	instrumental techniques based on visual observations of expressions (their position on the algebraic line, colour feedback…)	instrumental techniques available in Alnuset contribute to the conceptualisation	conceptualisation of the notion of function, equation and inequation
Uni Ap	TR (tree representation) in Aplusix	TR can be exploited as a tool of semiotic mediation for	structure sense of a numerical expression
Uni Cas	sub-environment Geometric Calculation of Casyopée	function as a tool of semiotic mediation for	mathematical meaning of function as co-variation mathematical meanings related to the processes characterizing the algebraic modelling of geometrical situation

Table 7. Instantiation of the CRQ key dimensions

V.1 The educational goals envisaged

We already developed some considerations on the educational goals when discussing the DDA&PP validation. Some of those considerations are still relevant, and we will develop them again. Thus inevitably the reader will find some repetitions.

V.1.1 Levels of specificity and articulations in the expression of the educational aims

Our initial remark is that educational goals are expressed through reference to mathematics or physics notions: function, vector, angle in 3D space, velocity… That is consistent with the priority grades assigned by the teams to epistemological concerns related to the identification

of educational goals when designing their TEs (see Del11): epistemological concerns were given high priority for almost each TE[4].

Even if teams shared epistemological concerns related to the formulation of educational goals, some differences appear among how those goals are (at least *a-priori*) specified and articulated.

In fact, as the above overview shows (Table 7), some teams investigated through their TEs the achievement of specific educational goals, whereas other teams investigated the achievement of general educational goals.

For instance, Unisi expressed the educational goals, pursued through their TE with Casyopée, as appropriation by students of "mathematical meaning of function as co-variation". Instead, ETL described the educational goals of their TE with Cruislet in more general terms, as students' construction "meanings about the concept of function"; analogously the educational goals of IoE TE with MaLT appear to be students making "meanings in relation to three dimensional geometry".

Alongside the variety of levels of specificity in the formulation of the educational aims, one can find a variety of levels of articulation of those aims. If one considers the just quoted TEs, one can find that Unisi further articulated their educational goals:

> "More specifically, as for the notion of function, students should consolidate or enrich:
>
> - the meaning of variables both geometrical and numerical,
>
> - the meaning of domain of a variable,
>
> - the meaning of function as co-variation over time (even when different kinds of variables are involved),
>
> - competencies related to the passage between different representations of function (at least, algebraic and graphical ones)"

On the contrary, neither ETL nor IoE tried to *a-priori* articulate the educational goals of their respective TEs.

A detailed articulation of their educational goals is also provided for other TEs. See for instance the specification of "structure sense" made by Unisi for their TE with Aplusix:

> "We adopt the definition of Hoch and Dreyfus (2006):
>
> 'A student is said to display structure sense for high school algebra if s/he can:
>
> - Recognise a familiar structure in its simplest form.
>
> - Deal with a compound term as a single entity, and through an appropriate substitution recognise a familiar structure in a more complex form.
>
> - Choose appropriate manipulations to make best use of a structure.' "

Analogously, MeTAH team formulated the educational aim of "understand[ing] the structure of algebraic expressions" for its TE with Aplusix, and specified the kinds of tasks which are assumed as indicators of students' achievement of this aim:

> "The students will be able to […]:

---

[4] Actually there is an exception: for their first TE centred on the use of Cruislet, Didirem did not formulate their educational aims in terms of mathematical notions, instead they expressed the aim of developing project teamwork. Consistently, epistemological concerns related to educational goals were not given high priority by the team for this TE.

- identify the form of an algebraic expression given in either of the following representation systems: tree, natural language, symbolic language;
- convert an algebraic expression given in one representation system into another one;
- solve problems involving algebraic expressions given either in natural language or in symbolic language (usual representation)."

The different articulations of educational aim is consistently reflected on the analysis of the TEs and is apparent in the answers to the Re-CRQs. For instance, if one considers Unisi analyses of their TEs, (s)he can observe that such analyses are focused on the "evolution of students' personal meanings" towards the designed educational goals. The students' personal meanings emerged are considered in the light of and related to such possible evolution.

On the contrary, teams having expressed more general educational aims, appear somewhat more receptive to catch different hints directly coming from the analysis of the data, as the analysis itself progressed. For instance, the aims of ETL TE with MaLT were to make pupils "construct meanings for the concept of angle in 3D space"; in the analysis of the TE they identified different clusters of meanings related to that notion.

> "The analysis of our data brought in the foreground the following three clusters of meanings constructed by pupils around the concept of angle.
>
> *Cluster 1: Angle as a slope while navigating the turtle in 3D space*
>
> <omissis>
>
> *Cluster 2: Recognizing (or conceptualizing) a dihedral angle in 3D space*
>
> <omissis>
>
> *Cluster 3: Angle as a dynamic entity for moving in different planes*
>
> <omissis>"

As clearly stated, this classification is made a-posteriori; it is a result of the TE. The three emerged clusters of meanings were not *a-priori* designed, and truly the planned didactical intervention did not aim at generating specifically those meanings. More precisely, there were not explicit desired meanings related to the notion of angle which should result from the TE.

Analogously, when analysing their TE with Cruislet, whose aim was students' "construct[ion of] meanings about the concept of function", ETL identified the construction of meanings relevant do notions related to the notion of function.

> "While students were interacting with the Cruislet environment according to the PP, several
>
> meanings emerged regarding the concept of function. We categorise these meanings in clusters
>
> that rely upon the concept of function. In particular, there are four major categories:
>
> 1. *Domain of numbers*:
>
> <omissis>
>
> 2. *Function as co-variation*:
>
> <omissis>
>
> 3. *Inverse function*:
>
> <omissis>
>
> 4. *Identity function*:
>
> <omissis>"

Here again, the identification of the emergence of meanings pertinent to the designed educational goals is made *a-posteriori* as a result of the TE analysis.

V.1.2 Definition of observables for attesting the achievement of the educational goals

Besides the different notions evoked in the specifications of the educational goals, a common aspect is that those goals are expressed by every teams as "meanings", "concepts", "understanding", "sense"… those terms are present in the formulation of almost any educational goals quoted in the previous section. To them, one can add, for instance: the "understand[ing of functions" and "the understanding of the idea of functional dependence in geometrical situations" (Didirem TE with Casyopée), and the construction of "meanings for the use of letters in algebra and the understanding of what algebraic expressions denote" (ITD TE with Alnuset).

The diffuse use of terms such as "meanings", "concepts", "understanding", "sense", may suggest a widespread attention by the teams for the cognitive dimension. This apparent attention seems to contrast with the fact that cognitive concerns about educational aims was not a common top research concern (§V.1.1 Del 11). Actually, a more careful reading of the TEs analysis show that the cognitive dimension is very often taken into account through the filter of a semiotic approach. That is consistent with the fact that semiotic concerns about educational aims were a common top research concern (ibidem).

The reference to "meanings", "concepts", and so on, raises the problem of evaluating the achievement of the stated educational goals. That issue is explicitly addressed by some teams, who recognized that personal meanings or concepts are not directly accessible. The acknowledgement of the inaccessibility of such dimension and the need to cope with it anyway, led some teams to explicitly express the need of specifying "observable elements" which could indicate the achievement of the envisaged educational goals and put into evidence the processes (cognitive, semiotic, social ones…) through which those goals were achieved.

With that respect, the emerging perspective appears consistent with a widespread perspective assumed in mathematics education which acknowledges the need for the researcher of defining a *corpus* of observables, in order to build a model of the individual's knowledge; such observables are the product of a *découpage* and an organization of the "real" made under a theoretical and methodological control, their relevance is determined by the researcher's *problematiques*[5] (Balacheff, 1994, p. 21).

We reported in the previous section, how MeTAH team articulated the educational aim of "understand[ing] the structure of algebraic expressions" specifying the kinds of tasks which are assumed as indicators of students' achievement of this aim. Students' performances in solving those tasks are seen as "observable elements" which can attest students' "understand[ing of] the structure of algebraic expressions". Also ITD articulated their educational goal "understanding of the structure of numerical expressions" (ITD TE with

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[5] « La prise en compte de l'élève a pour base des observables, produits d'un découpage et d'une organisation du "réel" sous un contrôle théorique et méthodologique dont la pertinence est déterminée par la problématique de l'observateur. Ainsi, le travail du didacticien comprend la constitution d'un corpus d'observables, parmi lesquels ceux appelés "comportements" de l'élève, à partir duquel est construit un modèle de connaissances appelé conception (Artigue 1990, pp.265-279). Ce modèle est une construction théorique du chercheur et non ce qui est effectivement "dans" la tête de l'apprenant. Sa validation expérimentale ne peut que confirmer sa valeur opératoire pour une problématique donnée, voire sa valeur explicative au sein d'un cadre théorique bien déterminé, mais elle ne peut lelégitimer comme reconstitution de la structure mentale effective de l'élève.» (Balacheff, 1994, p.21)

Aplusix) specifying those kinds of tasks which students are expected to be able to solve in order to be said to have achieved the educational goal: e.g. to represent numerical expressions as tree, to build trees for expression formulated in natural language, and to read expressions represented by trees.

As the respective analyses carried out by IoE and Unisi show, observables linked to educational goals are not only expressed in terms of tasks. IoE investigated, through their TE with MoPiX, "what concepts of motion were [are] represented through students' semiotic activity in the context of use of MoPiX". This question was articulated so to make clear the specific observables related to the educational goals: how "students operate in MoPiX with variables x and y, Vx and Vy, Ax and Ay", "what forms of language and other modes [do] students use to communicate about velocity and acceleration", "what choices [do] students make between and within the semiotic system offered by MoPiX", and whether and how the experience with MoPiX vary students' communications about velocity and acceleration.

Analogously, the Unisi addressed the question whether Casyopée could function as a tool of semiotic mediation for the envisaged educational goals (i.e. mathematical meaning of function as co-variation, and mathematical meanings related to the processes characterizing the algebraic modelling of geometrical situation), through investigating the possible collective or individual generation and use of artefact-signs, mathematical signs, hybrid signs or sentences, pivot signs, as well as of possible semiotic chains connecting those signs.

In all the above mentioned TEs, observables were *a-priori* defined as a result of an epistemological analysis of the educational goals or of the adopted theoretical frames.

In other TEs, observables are not made clear a priori. Nevertheless they clearly emerge from the TE analyses themselves.

For instance, discussing students' understanding of the notions of variable and parameter and development of competencies in distinguishing between them, Didirem (TE with Casyopée) mentioned as evidences of their achievements the following facts:

> "they [the students] had to learn how to animate parameters in order to find target functions by superposing the target's function and the associated function graphs.";

> "after the first two sessions, students were quite familiar with these functionalities of Casyopée, and never confused parameters and the function variable."

> "In the geometric part of the experimentation (lessons 3 to 6), students used parameters to treat generic cases (for instance a rectangle of size a and b) and had no particular difficulties with geometric constructions and expressions involving parameters."

The observables emerging from the analysis of Didirem TE may be expressed in terms of mathematical tasks. Instead, there is no reference to mathematical tasks in the observables emerging from the analysis of ETL TE with Cruislet. For instance, when discussing students' emerged meanings related to the "domain of numbers", ETL claimed that

> "Students navigating an airplane in the 3d map of Greece realized that the domain of the geographical coordinates is actually a closed group […]. The investigation of the range of the geographical borders as the domain of the function became the subject of study and exploration through the use of the DDA functionalities. In particular, students exploited the two different systems of reference and, experimenting with the values of the geographical coordinates, they define the range of the latitude – longitude values. This specific range of values has been considered as the domain of the functions according to which the displacements of the airplanes are relative."

The problematic relationship between "observables" and "meanings", "concepts", and so on is well expressed by ETL:

> "Although students didn't refer to the values as the domain of the function, we interpret their involvement in finding them, as a mathematical activity regarding the domain of the function."

Summing up, the comparison of the different TEs put into evidence a complex variety concerning how one can attest the formation, appropriation, construction of meanings made by students.

This variety reflects the variety of theoretical approaches through which semiotic concerns were filtered and made operative.

Despite the apparent heterogeneity, one can draw some common indications. As shown, the inaccessibility of that dimension is (more or less explicitly) acknowledged. This led teams to link educational goals to observable elements (which can be described in different ways): observables may be considered either as just indicators of students' achievements, or as true parts of what is called meaning or concept.

Finally, these elements are linked to the use of the DDA and the representations it provides.

V.2 The representations identified in the DDA

In the current section we discuss how, in their analyses, teams refer to DDA representations and link them to the achievement of the designed educational goals.

V.2.1 Different levels of specification of the considered DDA representations

One can notice that in the Re-CRQs there are different levels of specification of the considered representations (provided within the DDA). Some teams already specify in their Re-CRQs (or respective articulations) what the DDA representations in focus are. For instance:

> How does TR (tree representation) in Aplusix can be exploited as a tool of semiotic mediation for the structure sense of a numerical expression? […]

> (Unisi TE with Aplusix)

> How do students operate in MoPiX with the variables x and y, Vx and Vy, Ax and Ay? In order to achieve what goals?

> (IoE TE with Mopix)

> […] How do the distinct representations of parameters and variables affect the students' understanding of the two notions?

> (Didirem TE with Casyopée)

As one would expect, the focus on those representations is still maintained in the TE analysis:

> "The analysis of the protocols reveals that many students have internalized the recourse to the tree representation for comparing expression given both in SR [standard representation] and in TR [tree representation]."

> (Unisi TE with Aplusix)

> "Students' problem solving processes while using MoPiX consistently dealt separately with vertical and horizontal components of motion when adding and editing equations to models. This separation is supported by the structure of the semiotic system of MoPiX equations. Moreover, when using other modes of communication, students also described motion in terms of x and y components, making use of the terms Vx (or 'x velocity') and Vy and, to a lesser extent, Ax and Ay."

> (IoE analysis of TE with Mopix)

> "**As a first answers to Q2, we find**
>
> […]

- Distinction between variables and parameters
    - Seem to be good because of the different types of manipulations associated to each of the objects; in general there is no confusion between the manipulations and the meaning of theses manipulations in term of variable and parameters."

<div align="right">(Didirem TE with Casyopée)</div>

Other teams, instead, make a more general reference to the representations made available by the DDA. For instance ITD in their Re-CRQ refer to "algebraic representations mediated by Alnuset" and investigate their effectiveness for constructing meanings related to the use of letters, what expressions denote, and the notion of equation.

ETL generically refer to available representations, too.

> How do student use the available representations in MaLT to construct meanings for the concept of angle in 3d space?

<div align="right">(ETL analysis of the TE with MaLT)</div>

In both the TEs mentioned, the question by itself does not clarify whether according to the team there are specific DDA representations whose manipulation especially can support students' construction of meanings (or can mediate meanings) related to the targeted notions, or whether such meanings may be constructed through experimenting within the whole environment provided by the DDA.

In the TE analyses, there appears a more clear reference to specific representations and features of the DDA, and to students' actions on them, which are explicitly linked to students' achievements.

> "The algebraic line of Alnuset has been very fruitful because the dynamical management of the variable on the line has allowed students to assign meanings to the use of letters in algebra and to investigate what an expression indicates in an indeterminate way. By moving the variable x on the line students could easily control the values of an expression or could compare the value of more expressions, according to specific designed educational practices as described in the following sections."

<div align="right">(ITD TE with Alnuset)</div>

Certainly, there are still differences among the level of specification of the DDA representations considered, and their link with students' achievements.

There are also teams referring to DDA commands, features or environments, rather than to representations.

For instance, IoE investigated meanings related to 3D geometry constructed by students through their activity with MaLT, and in particular students' use of a specific feature of MaLT, namely the variation tool.

Analogously Didirem refers to specific features of Casyopée when studying how the use of Casyopée can affect students' understanding of functions geometrical computation

> "Q1: How does the geometrical computation followed by a free exploration of the situation in the dynamical geometry's window of Casyopée affect the students' understanding of the idea of functional dependence in geometrical situations?
>
> Q2: How does the computation of dependence implemented in Casyopée (several types of feedbacks linked to variables' choices) affect the students' competency to choose adequate variables in specific geometrical situations? […]"

Whereas Unisi refers more generically to a whole sub-environment of Casyopée, facing the question:

<div align="right">70/85</div>

> Does the sub-environment Geometric Calculation of Casyopée function as a tool of semiotic mediation for the mathematical meaning of function as co-variation?

Here again, even if there are some differences, in many TE analyses there appears the reference to the DDA representations and the link with students' achievements is more clearly discussed.

Concluding, it is worth noticing that the Re-CRQs raised in two TEs (ITD and METAH TEs with Aplusix) do not mention the DDA at all.

> "Is the educational activity based on the conversion among different representations (linear representation, tree representation, natural language) effective to mediate the understanding of the structure of numerical expressions?
>
> In particular,
>
> 1. Is this conversion among representations effective to teach how to represent a numerical expression as a tree?
>
> 2. Is this conversion effective to teach how to "build" a tree given a numerical expression or an expression described in natural language?
>
> 3. Is this conversion effective to teach how to "read" an expression represented by a tree?
>
> 4. Is this conversion effective to teach that there is only a linear expression for a tree while there could be different tree representations for an expression represented in linear form?"

> (ITD TE with Aplusix)

> How do the new semiotic register "tree representation" and its articulation with the usual and the natural language registers help the students understand the structure of algebraic expressions, […]?

> (MeTAH TE with Aplusix)

At a first glance, the lack of an explicit reference to the DDA in the Re-CRQs can originate a certain perplexity (in the context of our project). But this possible perplexity is certainly amplified because tree representation and standard representation of an expression are cultural mathematical representations also existing outside the DDA.

Actually, the TE analyses, respectively carried out by the two teams, put into evidence the importance of the representations (tree representation, and standard representation) as they are provided by the DDA.

V.2.2 On multiplicity of representations

There is an interest on the variety of representations provided in the DDAs, together with those present in more traditional settings, and in their interplay. This interest is coherent with what has been detected through the concerns analysis: we identified the "concerns about interactions between different representation systems within the DDA" as a high concern for most TEs.

This interest took different forms in the TEs.

ITD and MeTAH, following Duval, investigate whether conversions among different representations of (respectively) numerical or algebraic expression can "help students to understand the structure of […] expressions". The existence of different systems of representations for expression is thus explicitly taken into account in the formulation of the Re-CRQs, which question exactly how this fact may be linked to the achievement of the designed educational goals.

A different perspective is assumed by IoE, who through their TE with MaLT investigated (also) students' choices among different semiotic systems.

> "What choices do students make <u>between and within semiotic systems</u> in order to communicate their completed design to their peers? […]
>
> To what extent are students' <u>constructions in different semiotic systems</u> consistent with one another? In particular, are representations of properties of shapes consistent in different systems?"

It is worth noticing that ITD, MeTAH and IoE attention is not limited to the systems of representations offered by the DDA, but it also drawn to cultural and institutional ones, as testified by the high priority those teams assigned also to "concerns about interactions between different representation systems with institutional and cultural systems of representations".

Still another perspective is assumed by ETL in their TE with Cruislet. The existence of different systems of representation is mentioned in the Re-CRQ:

> "How students using spherical and geographical systems of reference in Cruislet construct meanings about the concept of function?"

The main attention is drawn on aspects different both from those considered by ITD and MeTAH, and from those considered by IoE. In fact, the focus is on the implementation in Cruislet of the link existing between the different systems of representations. Students' achievements are seen in connection with the fact that manipulations performed in a single system of representations, automatically bring to consistent changes in the linked systems of representation.

> "Cruislet was conceived as a digital medium for mathematical driven navigations in 3d large scale spaces. It is based on the idea of multiple linked representations (i.e., any action carried on a specific representation provides immediate change and feedback in all representations, Kaput, 1992). Students interacting with Cruislet environment could define the displacement of the avatar by employing either a Cartesian lat-long-height coordinate system or a vector-differential (fi, theta, r) coordinate system. Students exploiting the provided representations through the use of the DDA functionalities constructed mathematical meanings concerning the concept of function."

V.3 The relationship between the DDA representations and the achievement of the envisaged educational goals

As one could expect, there are many different perspectives from which teams investigated the relationship between the DDA representations and the achievement of the envisaged educational goals. Different perspectives lead to focus on different aspects related to the "processes" (including activities, actors, tasks, use of the DDA) which teams investigated to account the relationship between representations and achievement of educational goals.

Doubtless, each team took into consideration and specified the class organization, the activities and tasks to be accomplished, the actors' actions and roles, the use of the DDA, … when designing the Teaching Interventions. What is at stake now is whether and how those aspects are problematized in the formulation of the Re-CRQs and examined in the TEs analyses. This section aims at providing an overview of whether and how the teams explicitly investigated through their TEs different aspects regarding the relationship between the DDA representations and the achievement of the envisaged educational goals.

We start by briefly "comparing" the analyses (made by the experiment teams) of two distinct TEs: ETL TE with MaLT and Unisi TE with Casyopée. Through this comparison we mean to exemplify which aspects are in focus regarding the relationship between the DDA representations and the achievement of the designed educational goals. The discussion will be then broadened to involve also the other TEs analyses.

ETL reformulation of the CRQ is the following:

> How do students use the available representations in MaLT to construct meanings for the concept
> of angle in 3d space?

The process through which students achieve the educational goals is described as <u>construction of meanings</u>, and it is activated and fed by students' <u>use of the representations</u> offered from the DDA. The DDA offers the <u>context of use of specific representations</u>, students construct meanings through operating with these representations in the context of use provided by the DDA. Finally, the focus is apparently directed on the <u>individual aspects</u> of teaching-learning, in fact there is not any discussion about how interaction between the different actors or social activities in general support the individual's construction of meaning[6]. ETL examined neither the role of the teacher or that of the designed activities in feeding and fuelling the process of meaning making.

Those focuses are consistently maintained throughout ETL analysis of their own TE, as exemplified by some of the conclusions ETL drew.

> "student's active engagement to construct or to experiment with simulations of concrete objects
> that involve 'continuous' turning in the space seems to be related with the strong links between
> tool characteristics and educational goals with the given tasks"

The construction of meanings emerge as a result of students' active engagement to construct and experiment with the DDA. The process of meanings constructing is clearly linked to students' actions on the DDA, globally referred to as "experimentation"; and the idea of experimentation clarifies the expression "use of the representations". The focus on individual aspects of learning is confirmed too.

As for Unisi TE with Casyopée, the questions raised are whether "the sub-environment Geometric Calculation of Casyopée [can] function as a tool of semiotic mediation for the mathematical meaning of function as co-variation" and "for the mathematical meanings related to the processes characterizing the algebraic modelling of geometrical situation" respectively. As Unisi clarified, answering those questions requires one to investigate:

> 1. "the possible unfolding of the hypothesized *semiotic potential* of the artefact in relation to the
> designed tasks, and in relation to the target mathematical meanings"

That is the generation and use of signs related to the artefact and its use for accomplishing a task. The generation and use of those signs is stimulated through the requirement for students to produce written solutions of the tasks accomplished, written reports on the class activities and through the development of class discussions.

> 2. "the possible exploitation by the teacher of the unfolded *semiotic potential* for fostering the
> evolution of students' signs and meanings towards the desired mathematical signs and meanings"

For example, teacher's interventions to fuel the class discussions or her/his contributions to the collective generation of semiotic chains establishing connections between artefact-signs and mathematical signs.

> 3. "the possible evolution of students' personal signs towards the desired mathematical signs and
> the possible development of a texture of different meanings related to the target mathematical
> meanings which (that texture) contributes to enrich already formed personal meanings."

The process through which students can achieve the envisaged educational goals is described as an iterative cycle entailing: (a) <u>generation of personal signs,</u> - signs are related to the use of the DDA for accomplishing given tasks, and their generation is fostered by the explicit request to students to produce written solution of the tasks, written reports on the activities,

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[6] We do not mean at all that social aspects (interactions with the teacher or between pupils) are neglected, simply social aspects do not appear to be explicitly one of the main focuses of ETL study.

and by students' participation to collective discussion -, and (b) <u>evolution of those signs the desired mathematical signs</u> – this evolution should mainly result from students' participation to collective discussions orchestrated ad fuelled by the teacher. The generation of students' personal signs is not exclusively the result of the students' use of the DDA for accomplishing a task (though this use plays an essential role). Unisi consider equally crucial students' <u>engagement in different both individual and collective activities of production and transformation of signs</u>; and these activities are explicitly in focus in Unisi investigation. The resulting global perspective, which differs from ETL one, can be summarized saying that the DDA is considered as a tool susceptible of <u>mediating meanings</u>. Finally, as clearly emerges, the <u>social dimension</u> of the teaching-learning process is in focus too, and in particular the teacher's actions are explicitly investigated.

Those differences are consistently reflected in the specification of the concerns guiding the analysis process. In fact, as for the concerns related to the Modalities of Use of the DDA, Unisi and ETL shared concerns about the functions to be given to the DDA and their possible changes (c.2) and about the relationship between knowledge referred to the DDA functioning and knowledge referred to the educational goals (c.4). In addition, Unisi analysis was also guided by concerns about the tasks and their temporal organization (c.1), concerns about semiotic issues (c.3), and concerns about social organization and interactions (c.5).

In our opinion, the two discussed examples show quite well the diversity of focuses which could have been taken by the different teams in the analysis of their TEs.

Certainly, we are not claiming that the variety of focuses assumed by the two teams are antipodal to each other, and are not reconcilable. On the contrary we think that they can provide complementary perspectives all necessary to make sense of the complex processes under investigation.

Based on the results of the above comparison, we try to define a sort of grid (of possible aspects in focus) in which to place the TEs analyses. The aim is to highlight complementary perspective so to clarify how the different TEs analyses can contribute to provide a (necessarily general and generic) answer to the CRQ, addressed through the Cross-Experimentation.

V.3.1 Social aspects of the teaching-learning processes

As already mentioned, the social aspects of the teaching learning processes are valued by every team in the design of the teaching interventions[7], but this does not imply that social aspects are explicitly investigated through the TEs. With that respect, for instance, Didirem (in their TE with Casyopée) raised the question how the use of Casyopée affects students' understanding of functions. The social dimension is not addressed. Consistently social concerns (either related to the educational goals or to the modalities of use of the DDA) were not given high priority either in the TE design phase or in the TE analysis. The analysis carried out so far by the team confirms the main focus on the individual dimension of learning (i.e. students' personal engagement with the DDA) even if Didirem recognizes that

> "[A]ctivity of students can't be understood without considering the whole systemic environment (students, teacher, artefact, institution, cultural concerns…)"

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[7] We recall that Teaching Experiments entail both *teaching* – i.e. the design and implementation of teaching sequences or interventions, and *experimenting* – i.e. the investigation of hypotheses and the possible generation of hypotheses. When using expression such as "design of the teaching intervention", we mean to leave for a while research linked to the teaching intervention in the shadow. On the contrary, when speaking of "design of the teaching experiment" , we refer to both the dimension of teaching experiments: the didactical intervention and the linked educative research.

and consequently, the importance of analysing also the teacher's interventions.

On the contrary, IoE assign crucial importance in their study to the social dimension, such importance is explicitly recognized since the design of their TE with MoPiX (consistently social concerns were given high priority). Students' forms of language in communication are explicitly in focus. Communication is not just a means for accessing or evaluating students' achievements, rather it is through communication that meanings may be appropriated by students.

> "As students made use of various modes of representation ('everyday' language, specialist language of mathematics or of MoPiX, MoPiX programming, pencil and paper writing, drawing, MoPiX animation, gesture) different sets of semiotic resources and consequent meaning potentials were available to them."

V.3.2 Focus on teacher, tasks, activities…

There is a second dimension which is partly related to the possible consideration of the social aspects of teaching/learning discussed in the previous section, and along which the TE analyses and the answers to the Re-CRQs can be compared; it regards whether teams focus their TE analyses exclusively on students or also on teachers, on the kind of activities enacted, on the kind of tasks to be accomplished…

Some teams draw their attention especially on students; the specific tasks students are involved in, the teachers' actions, etc. remain in the shadow.

An example of that is provided by ETL analysis of their TE with MoPiX: neither the teacher' role, nor the tasks are discussed in the answer of their Re-CRQ, whose focus is indeed on students' (personal) construction of mathematical meanings about the role of equations:

> How do students construct mathematical meanings about the role of the equations while using the available representations in MoPiX to construct virtual models in the context of engaging in engineering design activities?

Actually, ETL reported excerpts of students-researchers interactions. But such excerpts are used to give account or illustrate students' achievements; there is not any analysis on the "effects" of the researchers' actions on students' construction of mathematical meanings.

Analogously, IoE attention is drawn on students. As previously emerged, IoE clearly consider the teaching/learning processes as a social process, and consistently communication is given great importance in IoE analysis of their TEs. Nevertheless, when analysing students' achievements and answering the Re-CRQ, IoE do not discuss aspects such as: teachers' provisions, elements of the chosen tasks… The emerging perspective is to examine (a) how students act on the DDA representations when working with the DDA (e.g. as for MoPiX, with variables x and y, Vx and Vy, Ax and Ay) and (b) how they communicate about their activity with the DDA. There is not the analysis of how the choice of the specific tasks with the DDA contributes to students' meanings making. Neither the teacher's role is discussed.

On the contrary the task is given great importance by Didirem in their TEs, at the extent that tasks are directly put under question in their Re-CRQs,

- both in their TE with Cruislet

  > Can we design tasks that a priori allow students with a very limited background in terms of vectors make sense of the complex semiotic system offered by Cruislet in such a workshop context, and use it for solving challenging and non trivial tasks involving 3D coordinates, vectors, displacements and curves, far beyond the curricular expectations at that grade?

- and in their TE with Casyopée

Q4 : How to built appropriate situations for exploiting the potential of Casyopée in that respect and what has to be the role of the teacher in the management of these?

Besides that, throughout the analysis of Didirem TEs one can find the reference to the specific task adopted, and the rationale for their choice:

"**The task organised** during the session 6 has been built to manage such movement between the frames and registers.

[…]

The designed tasks, the scenario and the DDA allow a great diversity of paths among these three poles, according to teams' choices. So the interplay between these settings and registers associated to windows changes may leads to conceptual development."

Other teams explicitly draw their attention to the teacher's actions (Unisi) or to the "educational practice" (ITD).

Unisi take explicitly into account and put under the lens how the teacher's actions contribute to students' achievements:

"In general, answering the question whether an artefact functioned as a tool of semiotic mediation for some mathematical meaning requires to investigate different but certainly related aspects:

[…]

the possible exploitation by the teacher of the unfolded *semiotic potential* for fostering the evolution of students' signs and meanings towards the desired mathematical signs and meanings;

[…]

In order to investigate whether and how the teacher exploited the unfolding of the *semiotic potential* for fostering the evolution of students' signs and meanings, we will focus on the semiotic actions which the teacher performs to orchestrate the class-discussions." (Unisi TE with Casyopée)

ITD, instead, investigate the effectiveness of the "educational practice" with respect to the achievement of the designed educational goals.

"Are these educational practices [based on integration between an algebra of quantities and an algebra of operations] mediated by Alnuset useful to construct meanings for the use of letters in algebra and to understand what algebraic expressions denote?

Are these educational practices mediated by Alnuset useful to construct meaning for equations?" (ITD TE with Alnuset)

Even if a definition of "educational practice" is missing, under that expression ITD apparently include the activities and tasks which students are required to accomplish; and ITD discuss how the accomplishment of those tasks is related to the achievement of the educational goals.

V.3.3 Construction of meanings and mediation[8] of meanings

The comparison between ETL TE with MaLT and Unisi TE with Casyopée suggested the appearance of at least two distinct possible global perspectives. The former perspective would lead one to consider the DDA as a tool offering the context of use of specific representations, through whose manipulation, exploration,… students construct meanings: in particular

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[8] As Hasan points out:
"the noun *mediation* is derived from the verb *mediate*, which refers to a process with a complex semantic structure involving the following participants and circumstance that are potentially relevant to this process: [1] someone who mediates, i.e. a *mediator*; [2] something that is mediated; i.e. a *content/force/energy* released by mediation; [3] someone/something subjected to mediation; i.e. the "*mediatee*" to whom/which mediation makes some difference; [4] the circumstances for mediation; viz,. (a) the means of mediation i.e. *modality*; (b) the location i.e. *site* in which mediation might occur." (Hasan, 2005).

students' achievements would be described in terms of meanings construction made possible through the use of the DDA (as exemplified in ETL analysis). The latter perspective would lead one to consider the DDA as a tool susceptible of mediating meanings: the use of the DDA for accomplishing tasks would lead to generate representations related both to the DDA representations or features, and to the targeted meanings which one desired to be mediated.

Even if the two perspectives are not antithetical to each other, they are certainly distinct. Nevertheless, it is not an easy task to identify which perspective is adopted in the different TEs.

This difficulty is partly due to the fact that "mediation", "construction", and related terms are widely used in mathematics education, but they are not always used as "technical" terms.

For instance, ETL conclude the analysis of their TE with Cruislet claiming that the DDA acted as a mediator:

> "we could say that the characteristics of the DDA, such as the visualization of the results of the objects' displacements on the map, acted as a <u>mediator</u> in students' engagement with the domain of function."

The term "mediator" appears to be used more as a math education common sense term with the generic meaning of "facilitator" than as a technical term. Consistently, throughout the TE analysis, the DDA is evidently considered as the environment in which students could experiment with the provided representations: through their experimentation students construct meanings related to the mathematical notions at stake.

In addition, even when used as a technical term, "mediation" can be given a wide connotation, which could sometimes result in a certain ambiguity. An idea of such a variety can be made when considering ITD TE with Alnuset. In fact the team manifestly gives a wide and rich sense to the expression "mediating role of the DDA", which is consistent with the adopted theoretical framework. For instance, the DDA is considered as mediating "educational practices", "representations", "variables, expressions, and propositions", "the development of the mastery of [these] capabilities and knowledge", "the development of control over expressions and propositions", "the construction of ideas".

The mediating role of DDA is put into evidence in IoE TE with MoPiX. IoE TE was meant to investigate the "concepts of motion represented by students […] in the context of use of MoPiX". In their analysis, IoE highlights that MoPiX provides students with an alternative language which become a crucial component for students to define and describe motion:

> "The oral language used by students through the course of the teaching experiment increasingly made use of component related terms derived from the MoPiX language.
>
> […]
>
> In the language of didactic functionalities, the characteristics of the DDA provide a representation of motion separated into horizontal and vertical components. This addressed the educational goals directly by <u>providing students with an alternative language</u> that moved them away from 'everyday' ways of representing motion, enabling analytic and quantitative approaches to defining and describing it."

Analogous perspectives are assumed in Unisi TEs, and MeTAH TE with Aplusix, and ITD TE with Aplusix.

Vice versa, ETL TEs and Didirem TEs describe the relationship between DDA representations and students' achievement in terms of construction of meanings which develops through the use of the DDA representations. For instance Didirem, who investigate "how the use of Casyopée […] affect[s] students' understanding of functions", during the TE analysis draw their attention especially on students' use of the DDA, in particular on the

changes between the DDA windows (geometrical, graphical and algebraic windows) made while solving tasks.

V.3.4 Analysis of students' actions: focus on production and transformation of representations.

In order to answer their Re-CRQs, almost every team investigated students' actions, including students' actions and interactions with representations. This kind of analysis is meant to shed light on students' achievements and on the processes trough which those achievements are attained (possibly relating the achievement with the use of the DDA).

Let us consider the following excerpt drawn from ETL analysis of their TE with Cruislet. ETL are discussing students' development of co-variation reasoning abilities.

"Initially most of the students expressed the covariation of the airplanes' positions using verbal descriptions, such as behind, front, left, etc. as they were visualizing the result of the airplanes' displacements. In the following episode students express the dependent relationship while looking at the result displayed on the screen.
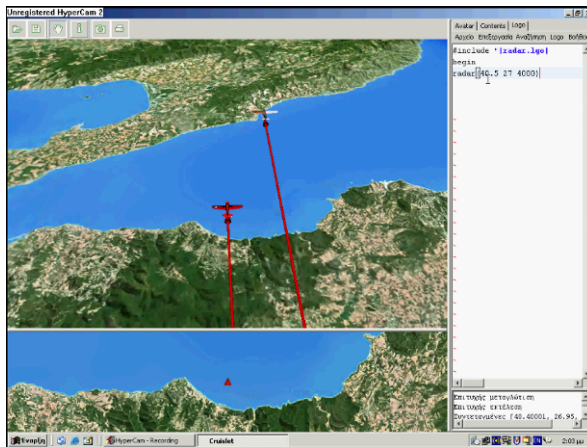
S1: So, he always wants to be close to us on our left.

R: Yes.

S1: And he is beneath, further down to us. Beneath.

S2: And behind.

Students experimented by giving several values to geographical coordinates in Logo and formed conjectures about the correlation between airplanes' positions. Through their interaction with the available representations, they successfully found the dependent relation of the function in each coordinate, resulting in their coming into contact with the concept of function as a local dependency. In fact, one of the teams conceived the relationship among each coordinate as a function, as it is obvious in their notes on the activity sheet.

> Our Lat is x, his Lat is x – 0.1
>
> Our Long is y and his is y – 0.05
>
> Our Height is ω and his is ω – 2500m.

It is interesting to mention that students separated latitude and longitude coordinates on the one hand and that of height on the other as they were trying to decode the hidden functional relationship between the airplanes' height coordinates."

Several kinds of students' actions and interactions with representations emerge from the reported episode: students' interpretation of the visual feedbacks provided by the DDA, students' experimentation with Logo procedures and command action, students' verbal productions (in interaction between them and with the researcher) and students' written productions.

Not all those actions are investigated by ETL. In fact, students' achievements are apparently considered as results of students' "interaction with the available representations". Verbal and written representations produced by students are treated as indicators of students'

achievements, and of the relation between those achievements and the use of the DDA. One may say that verbal and written representations produced by students are "observables" attesting students' achievements, and their relation with the use of the DDA. The process of production of representation is not taken into consideration in the process of construction of meanings[9].

Also Didirem analysis of their TE with Casyopée focuses on students' actions on the DDA representations: the analysis puts into evidence students' changes of Casyopée windows (geometrical, graphical and algebraic windows) made while solving tasks. With that respect, one can observe that Didirem attention for students' changes of frames and registers is not motivated by the consideration of students' production or transformation of signs as processes through which meanings are appropriated or generated, instead such attention is a result of the team's epistemological analysis of the notion of function, as explicitly declared:

> "Our **epistemological sensibility** about functions (Duval, Douady…) helps us to split the analysed situation into several domains of work
>
> - a geometrical frame
>
> - a functional frame which split itself into several registers of representations
>
>   - a graphical register
>
>   - an algebraical register."

A focus on students' actions on the available representations is not irreconcilable with a focus on students' productions of representations. The two can be seen to contribute to students' construction of meanings, as exemplified in IoE analysis of their TE with MoPiX and MaLT. In fact both investigated how students operate with and within the DDA, as well as how they communicate. The Re-CRQ addressed by IoE through their TE with MoPiX, was articulated also in the following questions:

> "How do students operate in MoPiX with the variables x and y, Vx and Vy, Ax and Ay? In order to achieve what goals?
>
> What forms of language and other modes do students use to communicate about velocity and acceleration as they work to construct animations and to interpret sets of equations and graphs?"

Both students' actions and productions of representation are seen as crucial contributions for students making meanings:

> "As students <u>made use </u>of various modes of representation ('everyday' language, specialist language of mathematics or of MoPiX, MoPiX programming, pencil and paper writing, drawing, MoPiX animation, gesture) <u>different sets of semiotic resources</u> and consequent meaning potentials were available to them."

IoE simultaneous attention for students' actions and speech is consistently witnessed by their multi-modal presentation of the data collected:

> "each transcription is structured to indicate simultaneously at any point in the extract: speech of the participants; interaction with MoPiX (e.g. dragging, editing, authoring equations, playing or stopping an animation); screen display; gesture; pencil and paper production (including use of computer based analogues of pencil and paper, e.g. Paint)."

Transformation of representations (conversion among different representations) are the processes in focus in ITD TE with Aplusix and MeTAH TE with Aplusix, who following

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[9] We are not claiming that ETL do not consider in general the process of production of representations as part of (or relevant to) the individual's construction of meanings, simply this possibility is not apparently investigated through their TE.

Duval consider the transformation as crucial in students' generation of meanings about mathematical objects..

Students' engagement in production and use of representation is considered to play a crucial role for the achievement of the designed educational goals even in Unisi TEs, to the extent of coining the locution "semiotic activities" to especially refer to the activities involving production, interpretation and transformation of representations (e.g. writing reports, participating in discussion)

> "TR [tree representation] has been an effective artefact to reach the envisaged educational goals, and in particular the structure sense. With this respect, fundamental roles have been played by
>
> o the individual reports, that have fostered individual reflections and the explicitation of personal meanings,
>
> o the classroom discussions, in which the teacher could promote the evolution from artefact-signs towards mathematical signs and mathematical meanings."
>
> (Unisi TE with Aplusix)

The discussed variety does not apparently reflect on the concerns. Actually, there exists a certain difference among the concerns inspiring the design (or guiding the analysis) of the TEs, but that difference does not appear strictly related to the issues discussed. On the contrary, for instance almost every team assigned high priority to semiotic concerns (related both to educational goals and modalities of use of the DDA) in design and in the analysis of the TE as well. Hence the highlighted variety is more likely related to the theoretical frameworks adopted. In fact one expects that high priority concerns are those for addressing which teams are especially equipped with theoretical frameworks. Sharing a concern does not imply sharing the lenses through which addressing it, that is consistent with the documented heterogeneity.

All that reveals different perspectives, different theoretical frameworks, different sensitivities which altogether contribute to shed light upon the complex system of processes underlying the meaning making through digital media.

VI. Synthesis and perspectives

As widely discussed in previous deliverables (Mariotti et al., 2007) the design and the analysis of cross experimentation had their roots in the two theoretical tools set up in WP1:

- the construct of Didactical Functionality,

- the system of concerns.

Both these constructs have been utilised in the elaboration of a common methodology for our experimentations that was based on making explicit a common characterization of "representation" and on formulating what we have called the Common Research Question.

Moreover the language of concerns had a very effective role in order to make explicit the general assumptions inspiring the design of the TEs, while the construct of the CRQ is going to play a crucial role in the recomposing of the contributions coming from the different TEs.


VI.1 Making explicit the educational goals

A first and immediate field of comparison between the different TEs is constituted by the set educational goals, in fact for each DDA it was possible to compare the EGs that each TE declared to be in focus and in respect of which the didactic success of the experiment was to be evaluated. Interesting to remark the proximity, if not even the coincidence between the EGs expressed by different teams for the same DDA.

Coincidence or proximity between EGs are certainly more evident because of the use of mathematical language in their expression. At one time, differences in the formulation of the EGs are clearly to be referred to the different Theoretical Frameworks adopted within which the formulation of the EGs themselves is framed.

In fact it may happen that mathematical themes or notions reformulated into EGs are expressed in a way that is consistent with a specific TF of reference. That means that the formulation of EGs is accomplished according to different conceptions of "learning", of "teaching", of what means "to have a certain knowledge or concept", and the like.

Accordingly, for each team one finds the use of different modalities for recognizing the achievement of a particular educational goal.
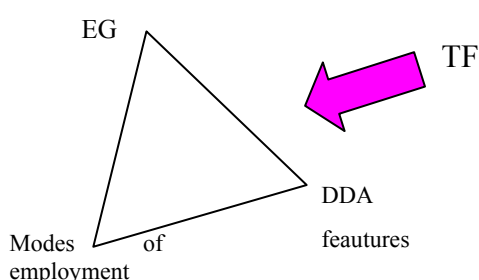
Differences in respect to epistemological and pedagogical approaches are clearly evident but, as expected, the constraint of using the categories of DF constitutes a strong element of uniformity. To use a metaphor, we can say that differences seem to be *filtered* by the frame of DF and because of this filter they assume a consistent form. Moreover the effort of making explicit assumptions and hypotheses and the corresponding theoretical elements allows one to control and compare such differences, as well to become conscious of them. In this deliverable the cross-analysis of the different TEs has just started. Further investigation is needed but we envisage a possible track in the analysis and comparison of EGs and their achievement with the aim of exploiting the filter effect of DF in order to harmonize differences and produce an articulated consistent answer.

VI.2 Different sensibilities to the Curriculum and its constraints.

The identification and formulation of the EGs also reflect how curricular constraints are taken in charge by the different teams: in fact, different sensitivities to curriculum and its constraints emerge.

This aspect has also a clear counterpart in differences in the nature of the DDA, its proximity to school practice both in terms of appearance of the objects the user has to interact with, the kind of tasks that the environment affords to propose, and the like.

Generally speaking, it is possible to recognize a tension between innovation and institution. What was already clear in the design of the DDA, has become evident in the experimentation. In fact, the integration of the teaching experiment in the sequence of the regular classes does not present uniformity. The tension between what is expected by the curriculum and what comes from the teaching sequence was solved by some teams in the realization of (part of) the Teaching Experiment outside the regular school time.

A DDA such Cruislet has a high intention of innovation that can make its use hard to be adapted to regular classes. But something similar can be found even for Aplusix. In fact, although Aplusix is to be considered quite close to institutional expectations, the particular feature of environment – the tree representation – that was in focus in the teaching experiment constitutes something unfamiliar and far from school practice.

VI.3 Representations and mathematical objects: Semiotic concern and its different articulations

According to the fact that the semiotic concern was a top common concern, all the TE have a common focus on the representation of mathematical objects offered by the different DDA and all the TE provide a positive answer about the potential offered by the different DDA in respect to the achievement of different learning goals.

Never the less the different theoretical approaches have different modalities to explain/describe the reference between the DDA – specific elements and specific way of using it – and the mathematical object.

Each team utilized different theoretical tools to approach the functioning of the representations available in a DDA (and the related semiotic processes) in relation to the achievement of the specific EG. In fact differences are remarkable and different levels of granularity are present in the description of the semiotic processes in play.

It seems possible to identify two main perspectives; it means a privileged focus on one element that does not mean a complete disregard of other elements. One perspective is centred on the subject/user, the student in this case, and on individual behaviour (actions on the DDA, verbal utterances in the dialogs with pairs and with the teacher/observer…) so that the relationship between the DDA and the mathematical objects is recognized and consequently observed in what happens when students approach a particular task. Evidence of meanings' construction is investigated looking at its emergence from solving a task.

Another perspective focuses on the social interaction between the students and the teacher considered as an essential element in the process of meanings generation. Accordingly, attention is brought (also) on how the relationship between DDA and mathematics emerges, and it is constructed, negotiated and shared through social interaction.

VI.4 Coordinating answers

Not all the theoretical frameworks utilized by the teams consider the same aspects of teaching/learning mathematics through digital artefact in the same way. Let us consider "semiotic processes" again: not all the theoretical frameworks have a model of the semiotic process available, and when it is the case, not all these models present the same elements and the same level of detail. In fact, even if all the teams declare a high semiotic concern, not all of them have specific theoretical tools for semiotic processes.

Although almost everybody talks about meanings, some of the research teams give only a generic reference to "construction of meanings", other approaches provide detailed descriptions of "evolution of meanings through specific semiotic processes".

In this respect comparison and coordination of results might appear difficult, but we see a possible track in providing coordinated answers to the CRQ at different levels of detail

For instance, the comparison between the teaching experiment carried out by ETL with MALT and the teaching experiment carried out by Unisi with Casyopée (described in chapter V) shows how the focus on meanings that students construct through their interaction with the DDA can be enlarged on the possible evolution that can result after the intervention of the teacher, in particular as a consequence of her intentional teaching action. In other words, the comparison highlighted the presence of different perspectives either in the design or in the analysis of the teaching experiments as well the possibility of their combination.

In fact these perspectives are compatible and we can see the possibility of their combination as well the emerging synergy.

For instance, it seems possible to carry out a cross-analysis of the data collected by different teams, combining the analysis from both the students' point of view and the teacher's point of view. On the one hand the analysis of teacher's action could highlight the role that she may have in the organization /orchestration of the learning environment.

On the other hand, the fine grain analysis of students' actions, as it is carried out by the ETL Team could offer a way to refine the analysis of the unfolding of the semiotic potential that is the main concern of the Unisi Team. In particular a more explorative approach, open to grasp whatever emerges from students' work, might highlight the emergence of students' interpretations that may diverge from what was expected and ask for specific teacher's intervention.

The results coming from such a cross-analysis would have the richness of the two approaches.

It is worth remarking the complementarities of these approaches. The former focused on the student a as an individual, interacting with the tool and for this reason constructing personal meanings related to this activity. The latter focused on the interaction between students and teacher and on teacher's intervention in making students develop such meanings according to specific EGs.

Although with some differences, a similar example of complementarities can be found in the comparison between the TEs carried out by Didirem with Casyopée and IOE with MoPiX. In this case it is still possible to recognize the complementary approaches: one based on a more exploratory perspective, aimed at observing and describing students action and interaction with the DDA, the other inspired by the objective of supporting students in the process of instrumental genesis.

In both cases a key element is the situation that is proposed to the students. In one case the explorative approach leads the researcher to propose tasks with a rather open request that let

students quite free in the solution process. In the other case, it is possible to recognize a careful design of the situation, where the sequence of the tasks, as well the single task, are attentively designed taking into account and controlling different didactic variables. In this case, in fact, the design of the situation has the objective of structuring and supporting the development of students' action according the specific EGs. The role of the task and the different potential that are related to the different choices directing its design is one of the main results coming out our comparative analysis of the TEs.

Though highly demanding, the coordination of the answers coming from different TEs seems to be rewarding in providing with new insight on the teaching/learning processes in relation to the use of technologies, and in establishing local networks of educational theories for approaching the study of meanings making processes through representing with digital media.

Certainly, the coordination of the answers and the recomposing of the different TEs contributions can be further developed. The possible tracks emerging till now constitute a strong suggestion for further investigation.

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