Casyopée

User Manual V 0.1

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ReMath Project. http://remath.cti.gr

Note

For this first manual, our choice has been to introduce the user by offering several situations with a step-by-step guidance. It will be complemented by a reference manual specifying the capabilities and options.

The first three sections introduce to the symbolic window pre-existing to the Dynamic Geometry extension. The two other sections explicate in details the capabilities of the Dynamic Geometry extension and the link towards the symbolic window. The last section illustrates how a function defined in the symbolic window can be handled as a geometrical object in the Dynamic Geometry extension.

Symbolic window (1) : Study of a function

This first example helps to become acquainted with Casyopée, and to quickly discover its main functionalities :

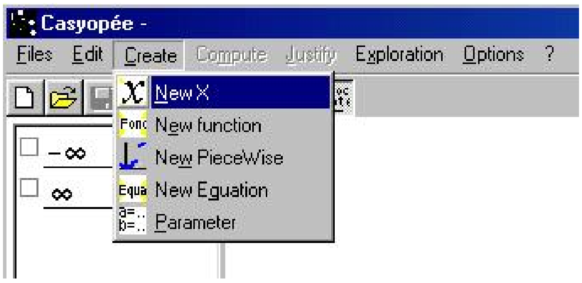
- creating an x-value to be used for a set of definition
- creating a function
- graphing it
- getting a table
- calculating symbolically
- solving an equation

More help is available through the help (?) command of the menu tool bar (now in French only).
This document refers to Casyopée's menu entries. Using buttons associated with the commands (see help) is also possible. The entries are written in **bold** font.

Preliminary notice

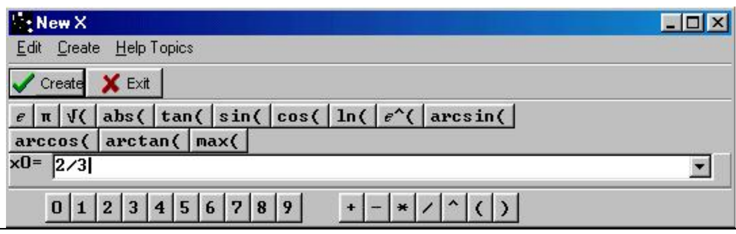
Casyopée records every creation, modification and result (of computations, of equations solving etc.) in the **Notepad** (bottom right). The user can add his own comments in the **Notepad** or remove a line. The **Notepad** can be displayed in full column using the button.

Let f **be the function defined on** $]-\infty\,;2/3[\ \cup\ ]2/3;\infty[$ **by** $f(x)=\frac{x+1}{3x-2}$

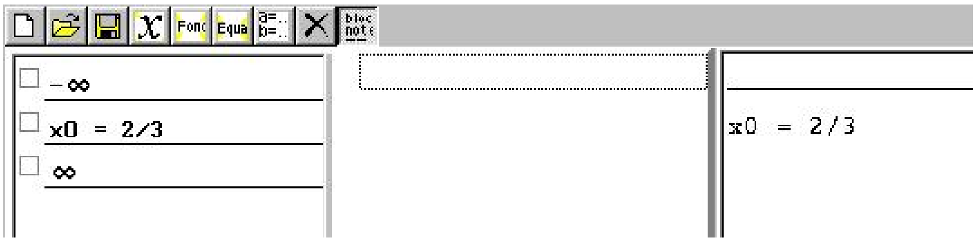


Creating an x-value

In the **Create** menu, choose **Create X**.

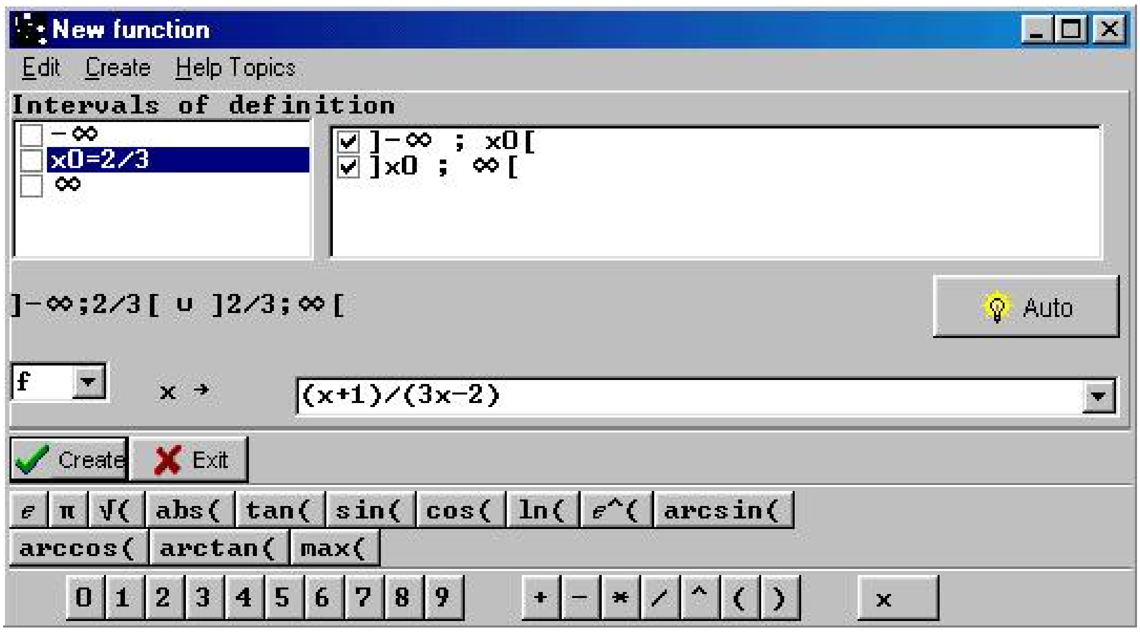
A dialogue box opens, enter the value, then click **Create** ; you can now enter a new value or quit the dialogue box by clicking the **Exit** button	

The new value appears in the window of x-values, top left, and is displayed like x0 = 2/3



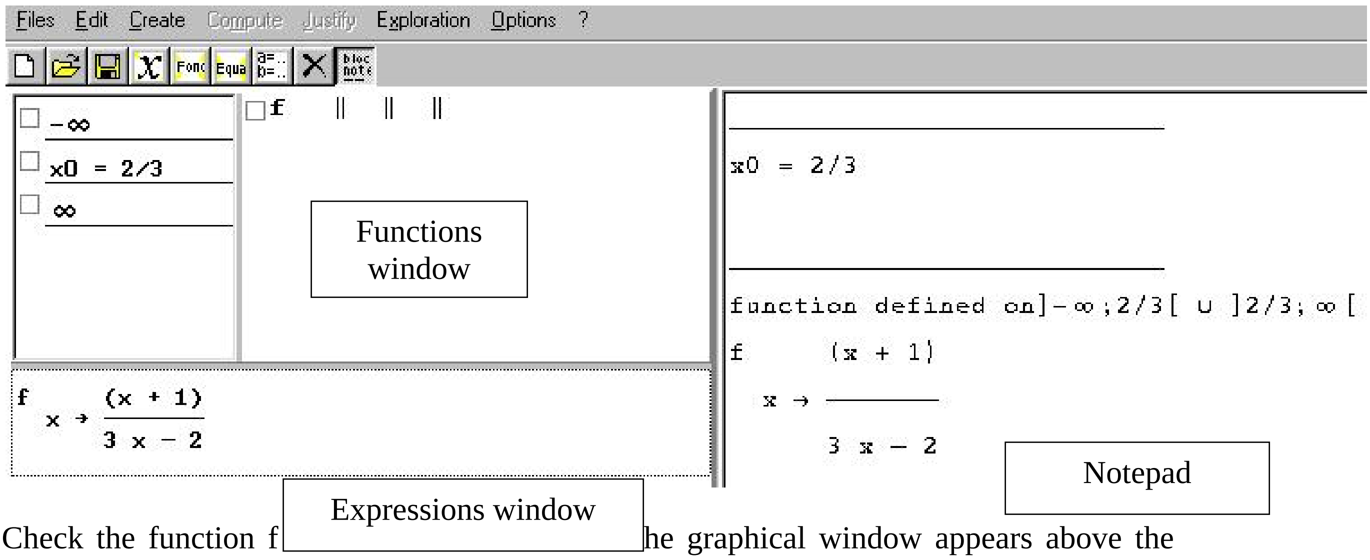
Creating a function

In the **Create** menu, choose **Create function;** enter the expression using keyboard keys or those proposed at the bottom of the box. Enter the definition set by checking intervals or x-values in the window at the top of the box.

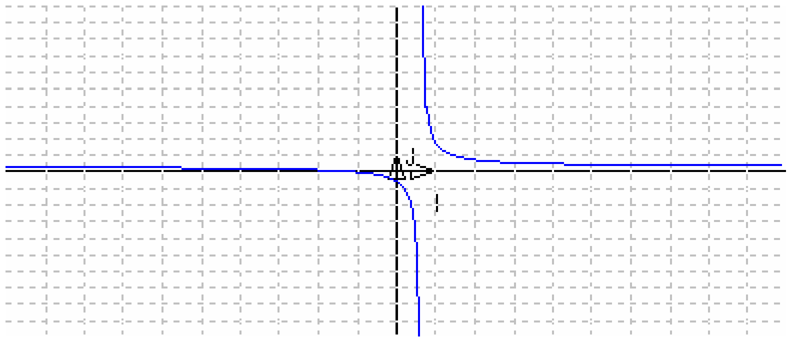


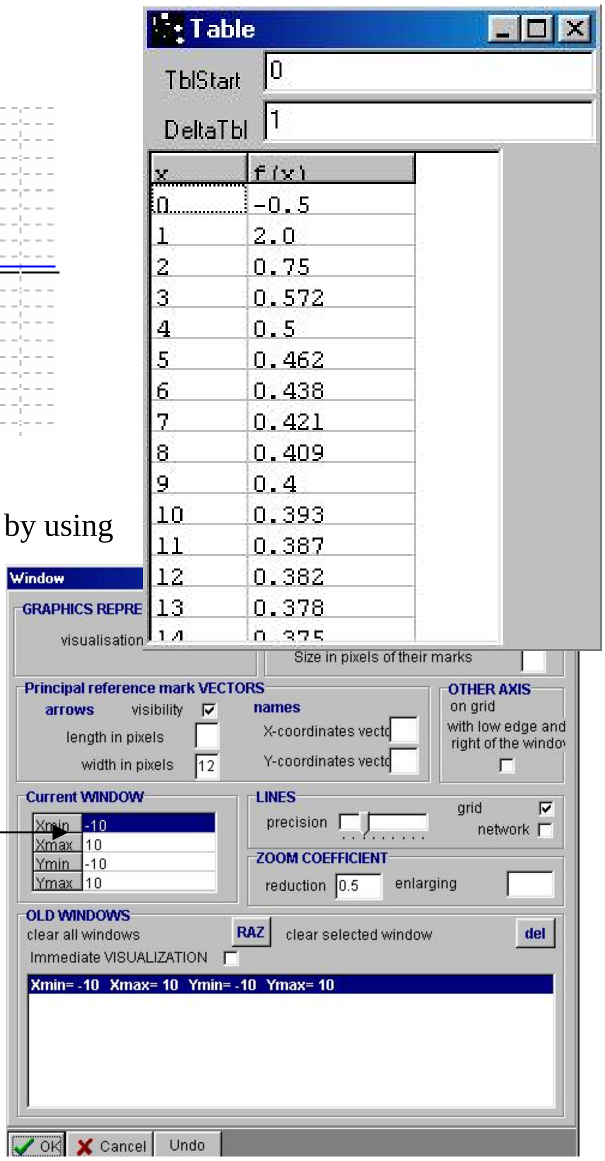
The **Auto** button gives the greater possible set of definition for the given expression

After the **Create** button has been clicked, the function and its expression are added in the respective windows. You can create several functions, and quit the dialogue box by clicking the **Exit** button.



Check the function f he graphical window appears above the Notepad.



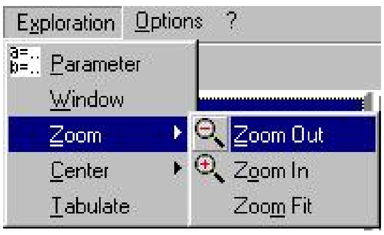


You may change the graphical representation of the function by using the **Exploration** menu or the window's toolbar.

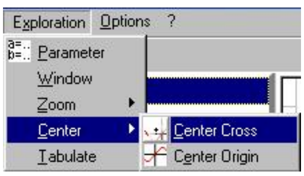
You can change:

- the **Window**
via a dialog box.

- **Zoom**.



- Center the cross or the origin **Center**.



- Obtain a table of approximate values **Table**.

Table

A table of symbolic values taken by the function for real numbers entered and checked in the window of x-values is also displayed above the graph. A contextual menu allows switching between values and limits.

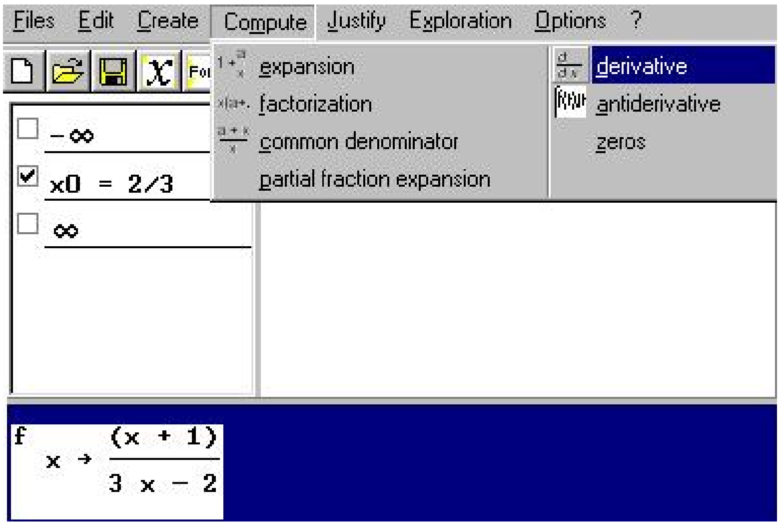
Table displays up to 15 decimal approximate values.

You can change the starting value **TblStart** and the increment **DeltaTbl**.

The Compute menu

In order to use the **Compute** menu

- select, with a click, the expression (it is highlighted in the colour chosen in the skin of your desktop),
- choose the computation to be made, the result is displayed in the Notepad and a dialogue box requests a confirmation that the x-values (resp expression) obtained are to be added to the corresponding window.

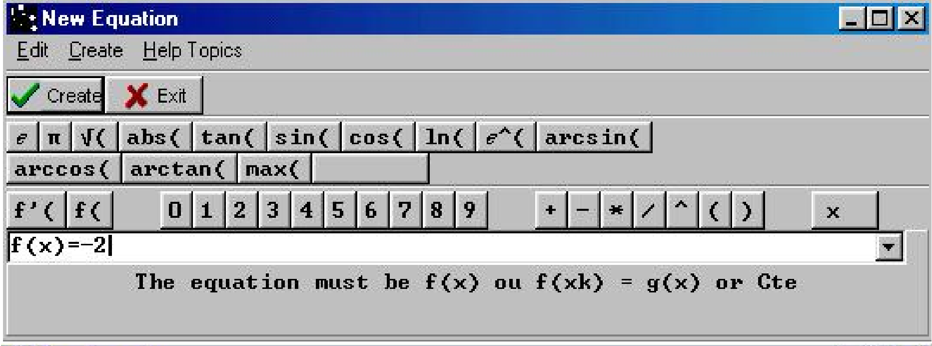


Choosing **derivative**, this confirmation is requested:

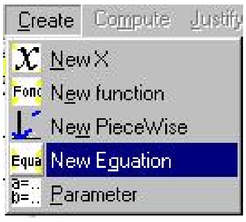
then an expression of the derivative appears in the expressions and functions windows, and in the Notepad

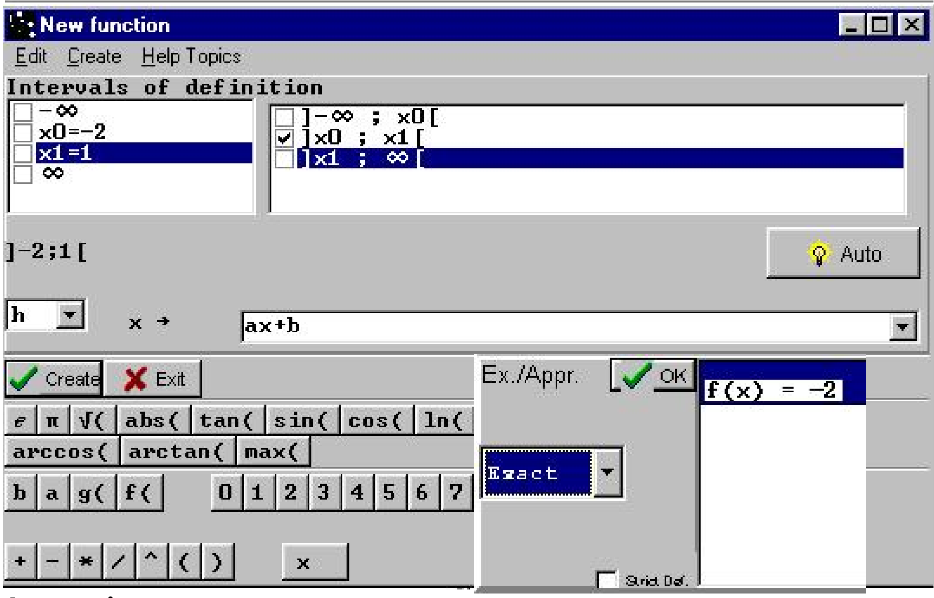
Expression window	Function window	Notepad
f $x \to \frac{(x+1)}{3x-2}$ f' $x \to \frac{1}{3x-2} - \frac{1}{(3x-2)^2}(3x+3)$	f ‖ ‖ ‖ f' ‖- ‖- ‖	$\frac{1}{3x-2} - \frac{3x+3}{(3x-2)^2}$ f' negative function defined on $]-\infty;2/3[\cup]2/3;\infty[$ f' $x \to \frac{1}{3x-2} - \frac{1}{(3x-2)^2}(3x+3)$

Solving an equation



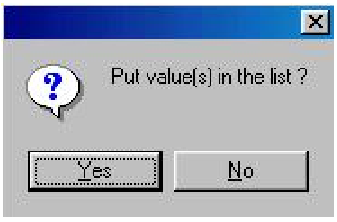
In the **Create** menu choose **New Equation**. Enter the equation and click **OK**.

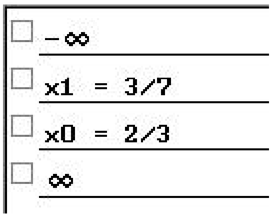




A new window opens bottom left: it is the equation window.

- Click on the equation (it is highlighted by the colour chosen for the skin of your desktop),
- choose in the list **Exact** or **Approximate**.





The solutions are displayed in the NotePad and a dialogue box asks whether the solutions should be placed in the list of values of x.

By clicking on "Yes", you make the solution appear as x1 = 3/7 in X-values window.

This the end of the first example Using the button, the **Notepad** is displayed in full column. It is easier then to read the results of the user's study. Saving is possible, Casyopée creates « .txt » files (text format).

Activate **Files** / **New** in order to open a new work sheet and to start the next example **Getting Started (2)**

Symbolic window (2) :

Study of a function with parameters

This second example helps to become acquainted with the functionalities of Casyopée for functions with parameters :

- defining a function with parameters
- animating and "unanimating" a parameter
- substituting values of parameters
- writing algebraic conditions

Preliminary notice

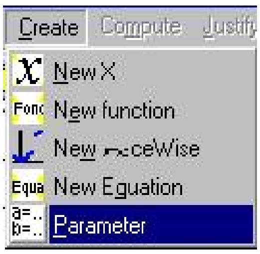
The buttons **OK**, **Exit**, **Cancel** or **Evaluate** close the dialog boxes.

Three functions *f*, *g* and *h* are considered :
***f* is defined on] – ∞ ; – 2] by $f(x) = -(x+2)^2 - 1$;**
***g* is defined on [1 ; + ∞ [by $g(x) = (x-1)^2 + 3$;**
***h* is defined on [-2 ; 1] by $h(x) = a\,x + b$;**

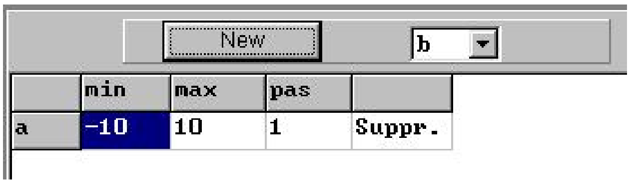
The goal is to find values of a and b to connect the three graphs continuously.

Starting

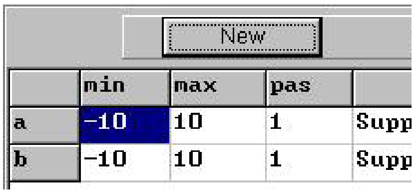
- Create the x-values -2, 1 (as in first example) ;
- Create the functions *f*, *g* (as in first example), checking the correct sets of definition.
- This how to Create the *h* function :
  in the Create menu, choose Parameter, then New so that the parameter **a** appears.



It is possible to change minimal and maximal values and the increment, and to delete the parameter.

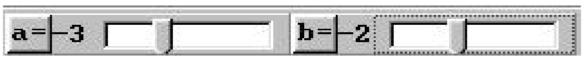


Please create also the parameter **b**:

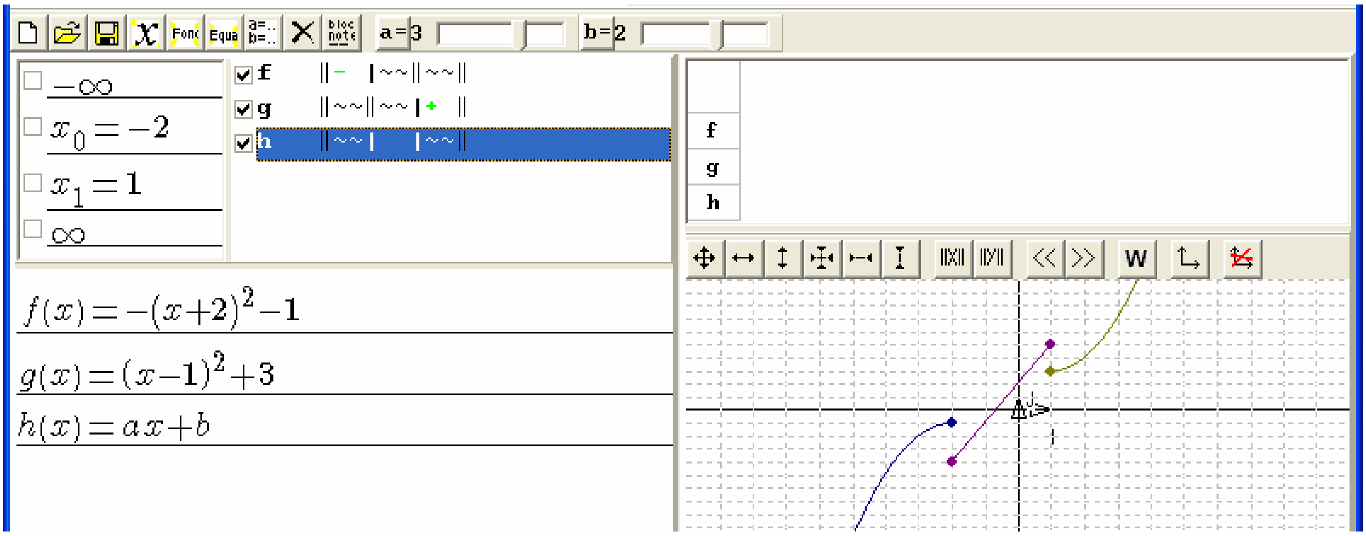


You can now enter the *h* function.

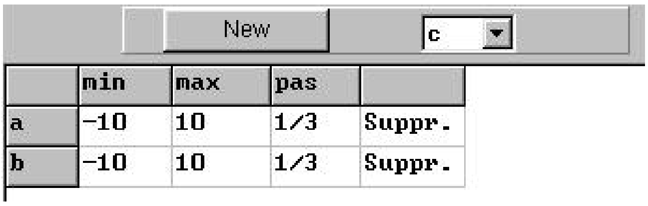
Animating parameters



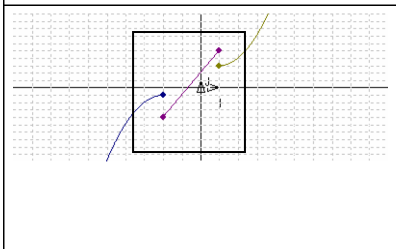
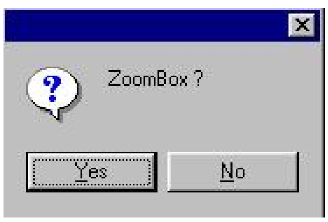
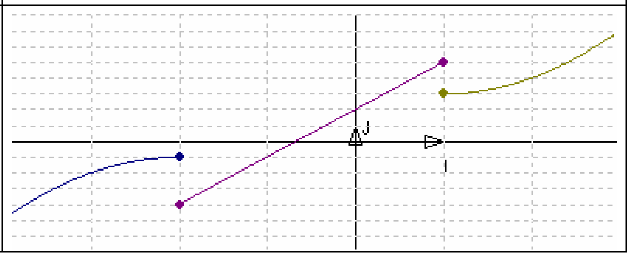
- You can instanciate parameters by clicking on them in the toolbar.
- You can animate the parameters with the mouse or the keyboard's arrows.
- Values taken by parameters are in the] min ; max[interval.
- Checking *f*, *g*, and *h* in the functions window, you should get a screen like this:
- The graph of *h* is updated according to values of **a** and **b**.



To update the increment of the parameters, select **Parameters** in the **Exploration** menu, then in the **Parameter** window change the increment.

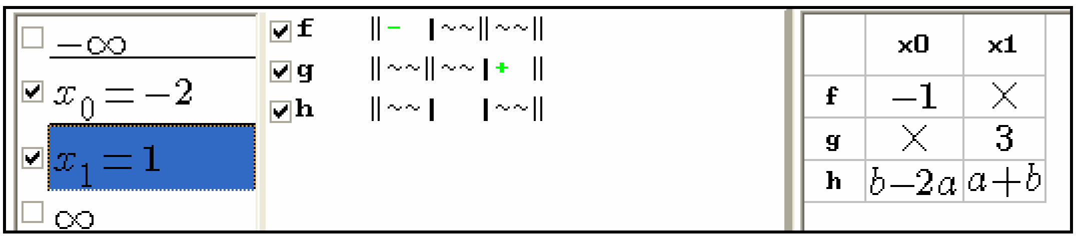


When the mouse cursor is in the graphical window, it appears as a cross, and you can use the Zoom functions:

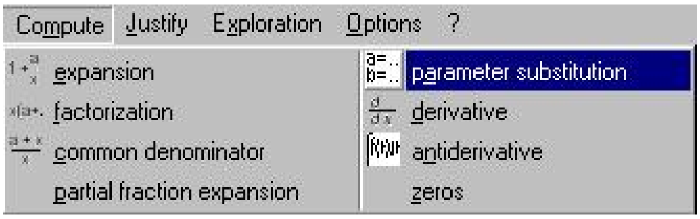
You can draw a rectangle, dragging the mouse right button.	You will be asked for a confirmation	The graph is expanded in the rectangle.
		

<u>Use of the symbolical values' window to get the conditions at [– 2 ;1] interval's bounds.</u>

Check x0 and x1 in the x-values' window. The symbolical values' window helps to express algebraically the conditions :



These conditions are **b** – 2**a** = – 1 et **a** + **b** = 3 ;
By solving this system, you can obtain suitable values for **a** and **b** (4/3 and 5/3).
Animate the parameters to give them these values and check on the graph.

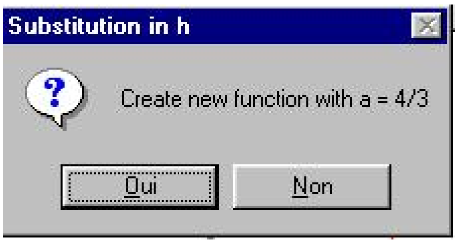


Parameter substitution

Select *h* in the expressions window.

With the **Substitute parameter** entry of the **Compute** menu, a new function is created where the values of **a** and **b** are replaced by those shown in the row of parameters values.

Confirmation is asked for each parameter:



A new function *h0* independent of the parameters is created andappears in the windows.

This example demonstrated the capabilities of Casyopée for solving problems with parameters.

In **Getting started (3)**, the **Justify** menu will be explained.

Symbolic window (3): Using the Justify menu

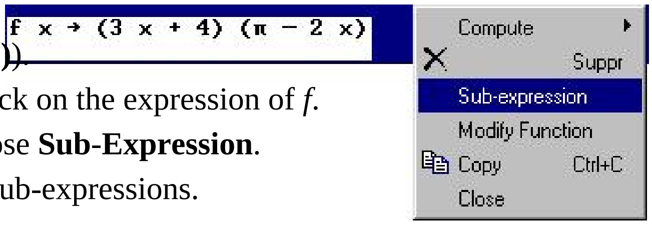
This third example helps to become acquainted with the justification part of Casyopée :

- How to justify the sign of a linear function
- How to use the rule of signs for products

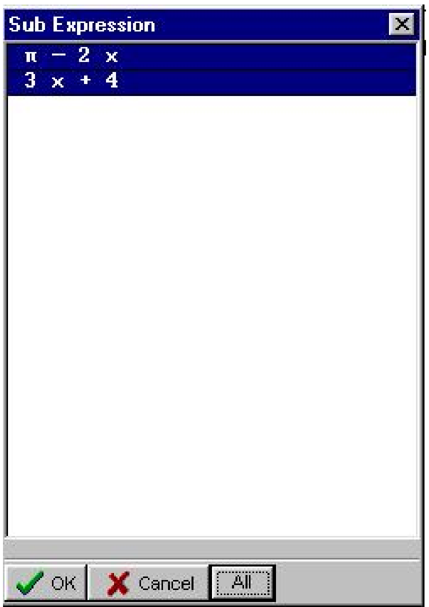
The sign of the function f(x) = (3x + 4) (π – 2x) is to be determined.

Two ways for determining the sign of a linear expression are demonstrated:
- **one uses results about linear expressions' sign (method 1),**
- **the other uses results about variation of linear functions (method 2).**

The conclusion will be about using the rule of signs of a product or of a quotient.



Enter the *f* function (see **Getting started(1)**).
In the expressions window, please right click on the expression of *f*.
In the contextual menu which appear, choose **Sub-Expression**.
The following window appears, it list the sub-expressions.



The selection of list's elements is as in Windows™ explorer :

- keeping « Ctrl » key pressed, click on non-consecutive elements on order to select them ;
- keeping « Shift » or ⇧ key pressed, click on first, then on last element of the list to be selected ;
  The **all** button selects all the elements.

In this example choose **All**, then click on **OK.**
The expressions and functions windows are completed by the selected elements named f0 and f1.

expressions window	functions window
f x → (3 x + 4) (π − 2 x)	f
f0 x → π − 2 x	f0
f1 x → 3 x + 4	f1

Sign of* (3*x* – 4) *with method 1

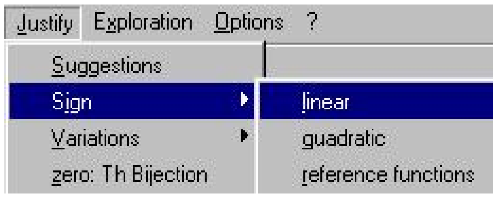
Highlight f1 in the **functions** window as seen above.

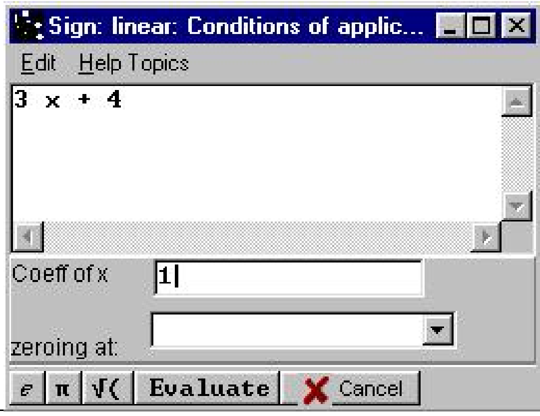
In the **Justify** menu, choose **Sign** then **linear**.

This dialog box appear :

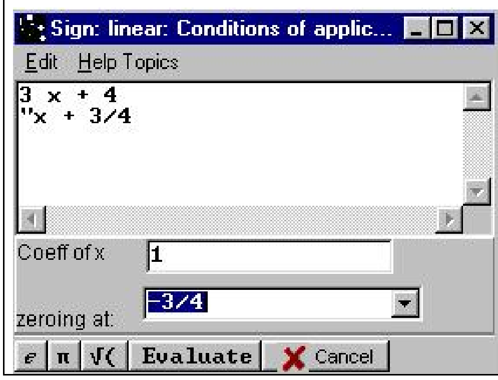
Complete **Coeff of x** and **zeroing at**.

Then click on the **Evaluate** button.

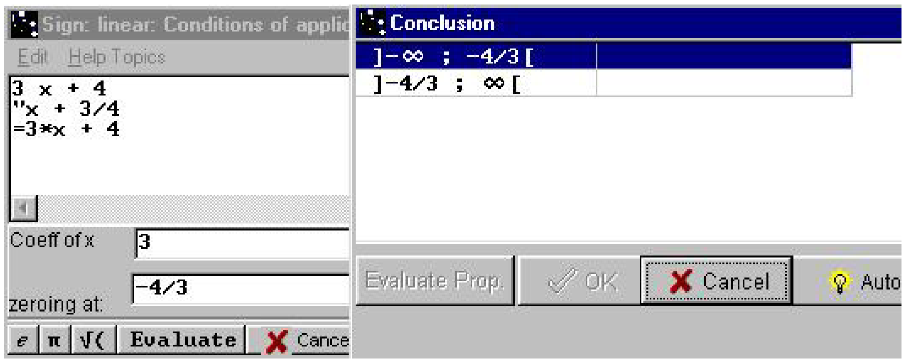




If there is an error, a line shows up the difference with the expression, you need to update the two inputs' value

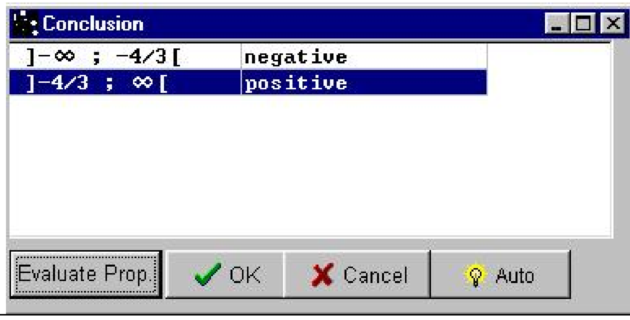


If the correct values are entered, after clicking **Evaluate**, a new window opens.

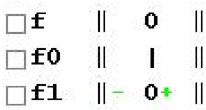


On each line, successive clicks make **negative** or **positive** appear; evaluate your propositions by clicking the **Evaluate Prop** button

When propositions are correct, the **OK** button is active, please press it.

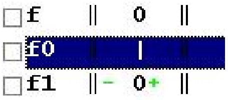


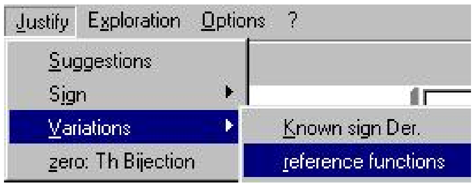
In the functions window, the results appear as a table of signs :



__sign of (π – 2x) with method 2__

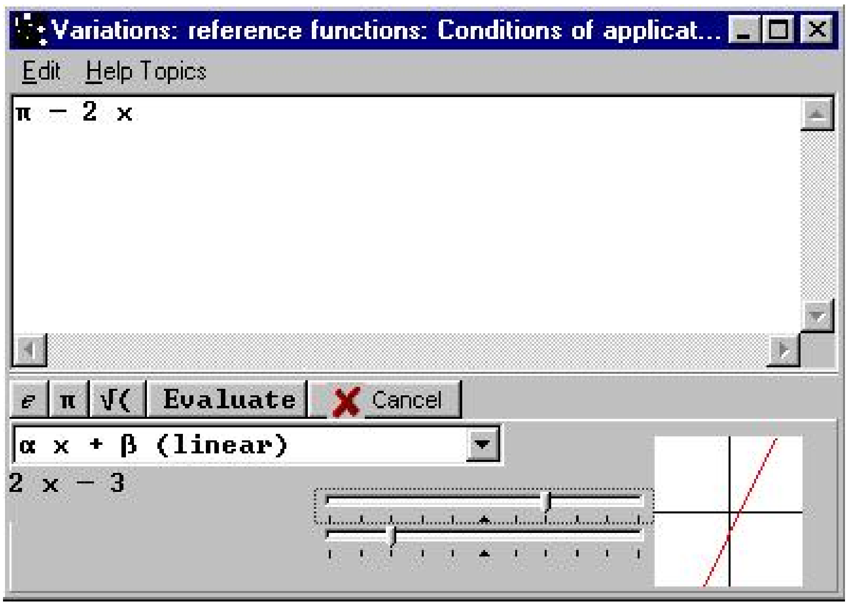
In order to determine the sign of (π – 2*x*), create the zero of the expression in the X-value window (see **Getting started (1)**) then select f0 in the functions window.





In the **Justify** menu, choose **Variations** then **reference functions.**

This window opens:



You can choose the type of function in the list, and you will get a graphical representation of the chosen function's type.

Click the **Evaluate**. button

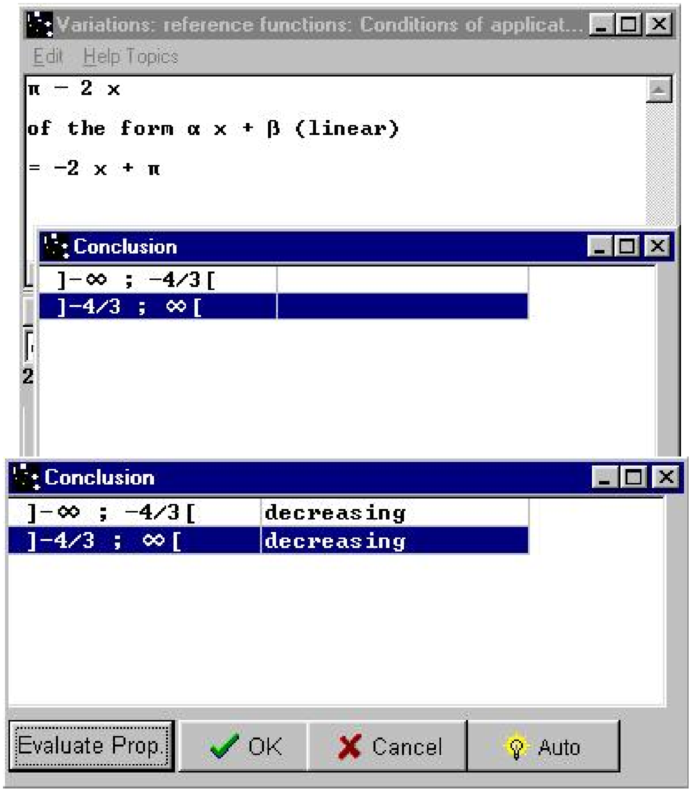
Two lines are written, confirming or not a correct choice;

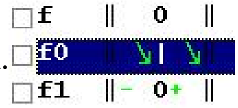
after a correct answer, the **Conclusion** window opens.

For each line, each interval, click in order to make **decreasing** or **increasing** appear.

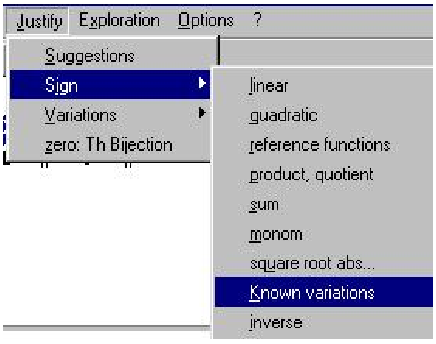
Evaluate your propositions with the **Evaluate Prop** button.

When propositions are correct, the **OK** button is active, please press it.

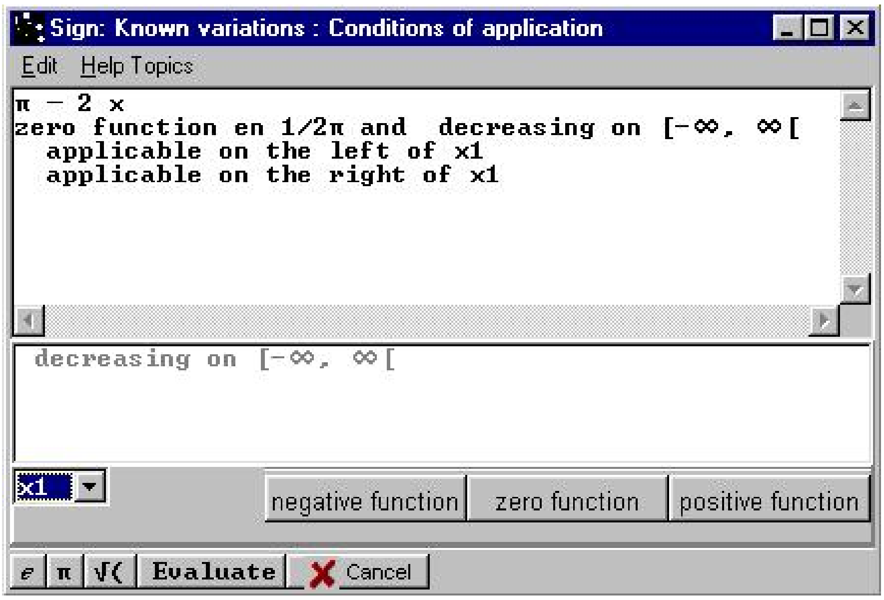


In the functions window, results appear as a table of variation. 

After variations, it is needed to justify the sign ; in the **Justify** menu, choose **Sign** then **Known variations**.

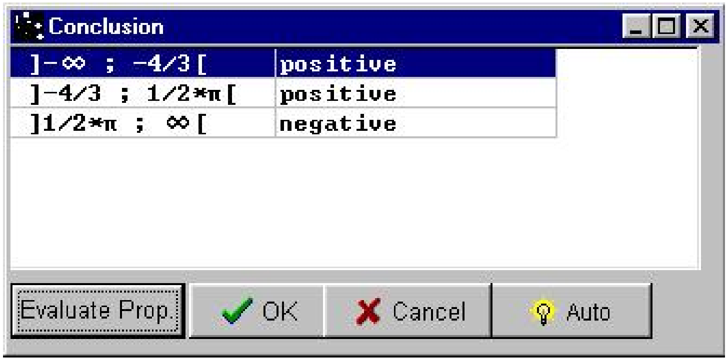


The following window appears:



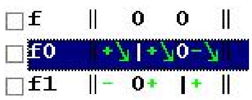
Firstly, select the value which makes the expression null, then click on “**zero function**” and on **Evaluate.**

At the top of the window appear the conditions of application needed for the sign determination.



After clicking on **Evaluate**, the **Conclusion** window appears, you need to click on the lines to make **positive** or **negative** appear ; then click on **Evaluate Prop.** in order to evaluate your entries ;

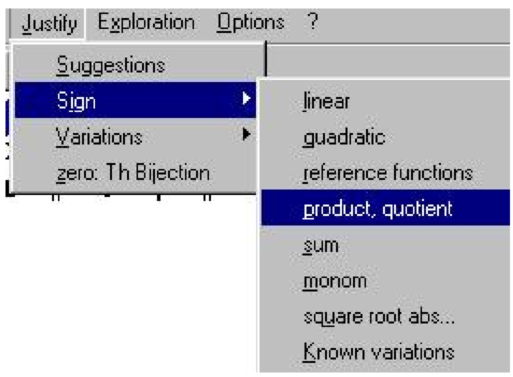
When propositions are correct, the **OK** button is active, please press it.

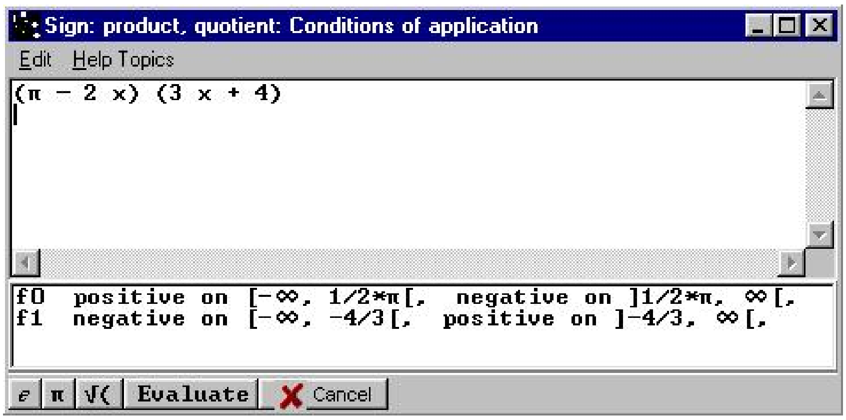
Results appear in the functions window, on the f0 line. 

Determine the sign *of* f(x) = (3*x* + 4)(π – 2*x*)

At last, in the functions window,
select f, and then in the **Justify** menu, choose **Sign** then **product, quotient**,

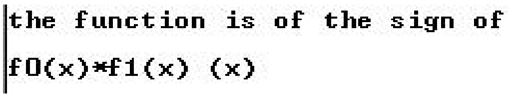
The following window appears:



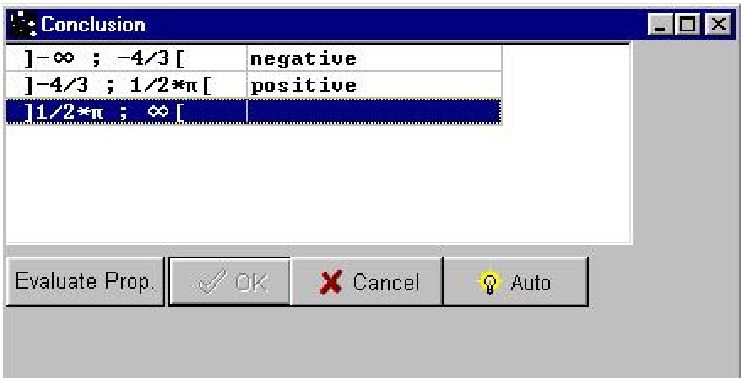


At the bottom of the screen select the two lines, because f is the product of f0 and f1.

Click on **Evaluate,** these two lines appear in the window :



Then the **Conclusion** window opens above :



The meaning of this window have already been explicated.

After pressing **Evaluate Prop**. and **OK**, the results appear as signs in the functions window :

Casyopée Dynamic geometry extension (1): An optimisation problem

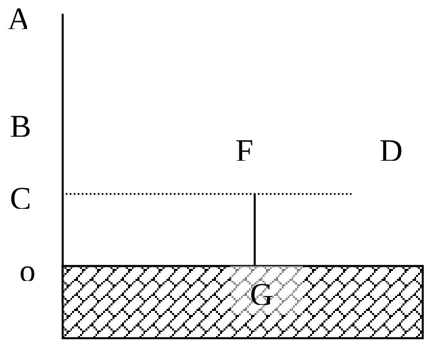
It is recommended that the user looks at the document “Getting Started with Casyopée” document available on the ReMaths project website remath.cti.gr. The document “Getting Started with Casyopée” will help a non-French speaking reader to get a basic idea of Casyopée's aims and functionality before the implementation of the extension to Casyopée.

In the following example we study the dependence between a geometrical calculation and a geometrical variable by optimising a distance.

The Problem

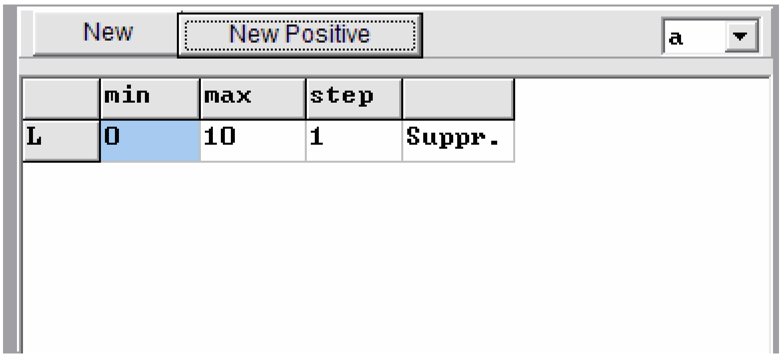
A farmer wants to enclose a stock pen of sheep and goats with a high fence to keep out the hyenas and jackals. The problem is he only has a limited length of suitable fencing wire (lets assume the length of the wire is L). He wants to have a rectangular enclosure, but he only needs to use the fencing wire on three sides of the rectangle as the fourth side is enclosed by the wall of his home (here we assume the x axis).
He extends his wire perpendicularly to the wall (here he follows the y axis from the origin o). The point o is one end of the wire and the point A is the other end. The point B is at the middle. The farmer then chooses a point C between o and B. [oC] will be the first side of the rectangle. Then the farmer extends the rest of the wire parallel to the wall (the x axis). The point D is now at the end of the wire. The farmer chooses a point F on the wire in order that DF=oC. He extends this length of the wire towards the wall, perpendicularly. The end of the wire is now on the wall (the x axis) at point G. The rectangle is oCFG.

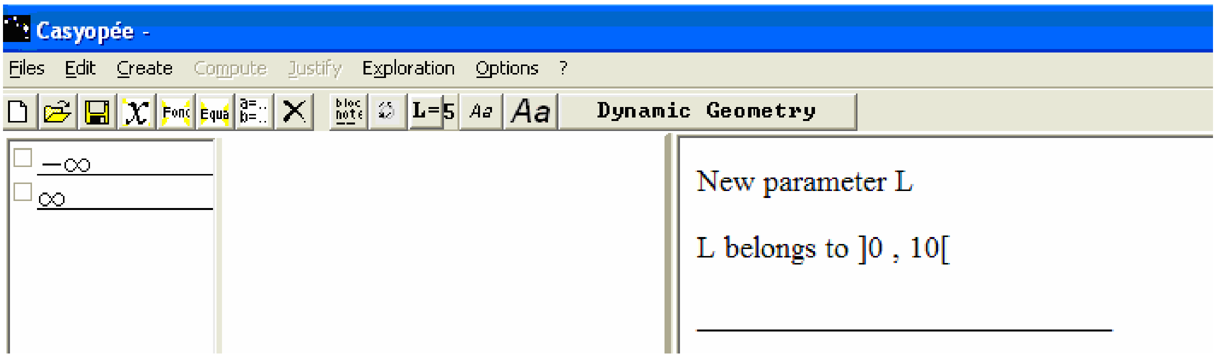
What position of C will give the farmer the maximum area for the enclosure oCFG?



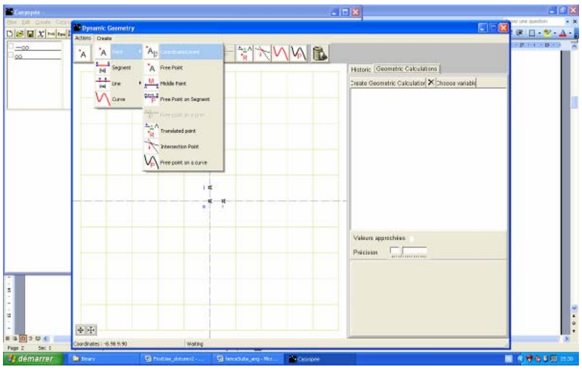
Creating a positive parameter

Create the parameter L in the main Casyopée window:





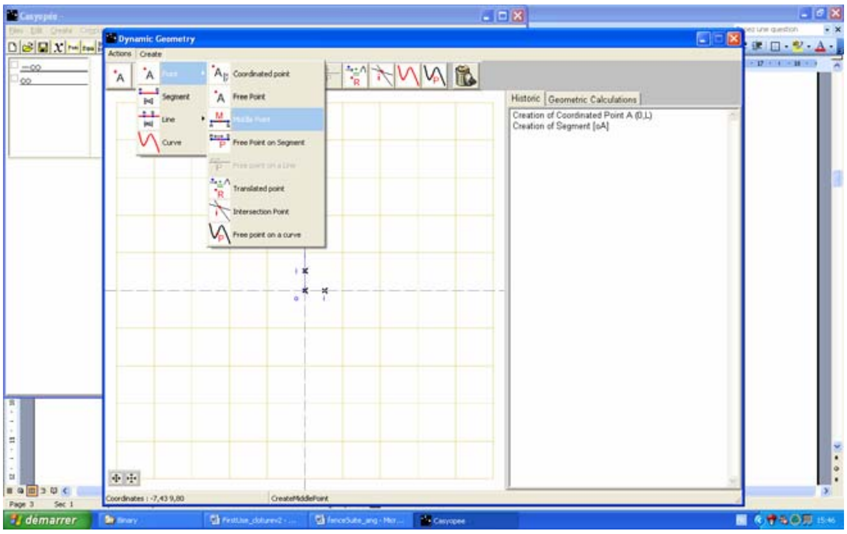
Switch to the *Dynamic Geometry* window, by cliking the corresponding button in the tool bar. Construct the rectangle oCFG by first creating the fixed point A(0,L) by either selecting *coordinated point* from the dropdown menu as follows:

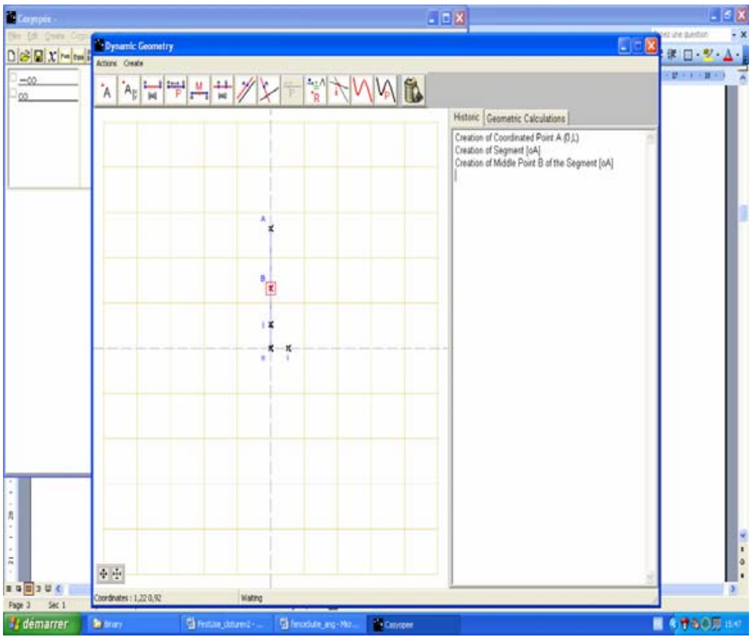


or selecting the button for the *coordinated point* from the tool bar:

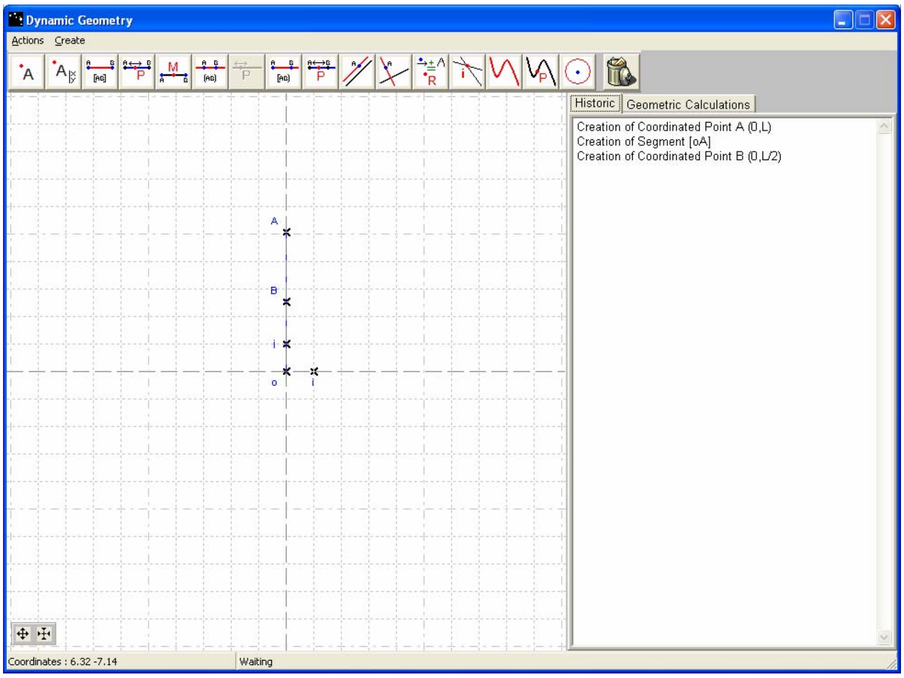
Next create the segment [oA] by selecting *Segment* from the dropdown menu of the *Create* option and then selecting *o* as the first designated point and then *A*:

Next create the point B(0,L/2) the middle point of the segment [oA] as follows:



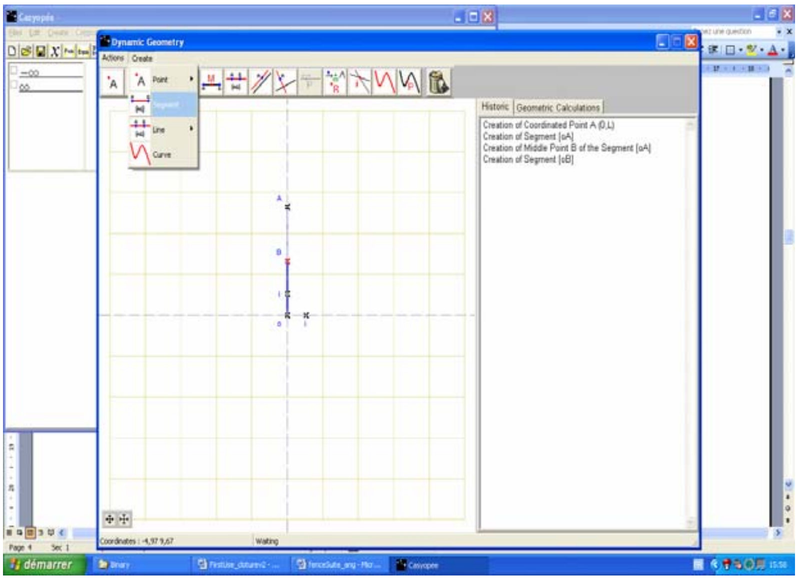


Or as the *coordinated point* (0,L/2)

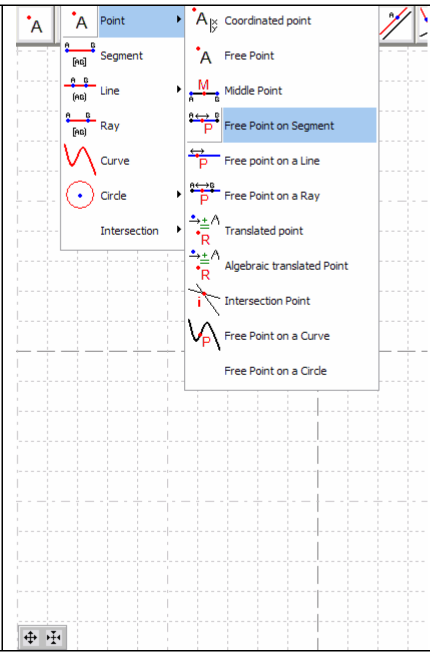


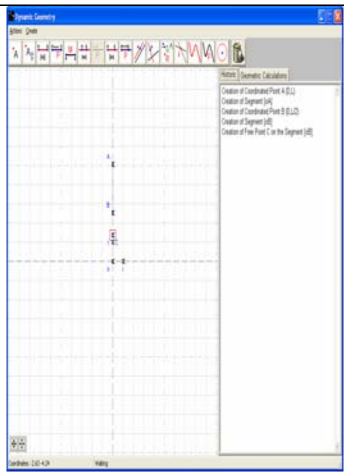
Creating a rectangle

Create the segment [oB] either from the dropdown menu as shown or from the toolbox menu



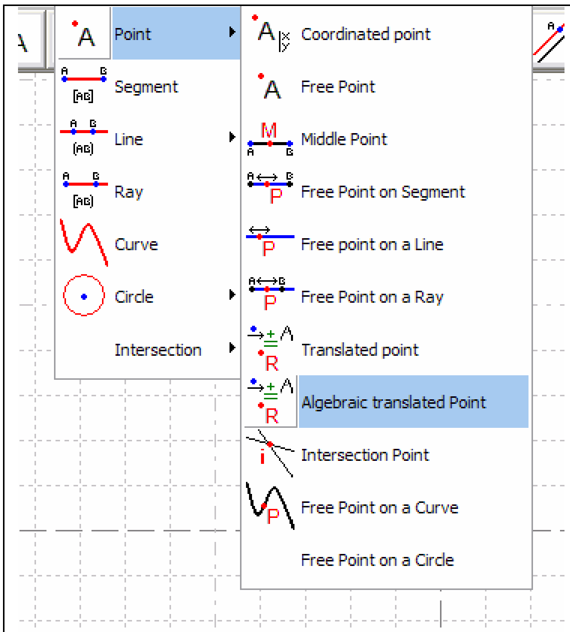
C is a free point on the segment [oB] created as on the right.
Be careful to select *Free point on the segment* and not *Free point*, in order that the point stays on the segment and cannot be moved outside.

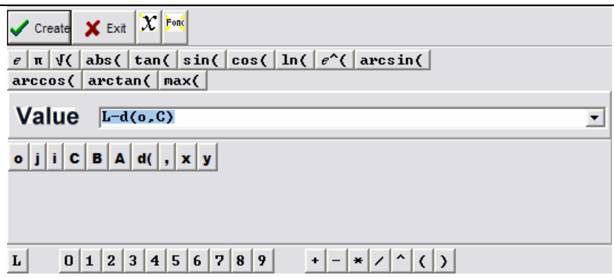


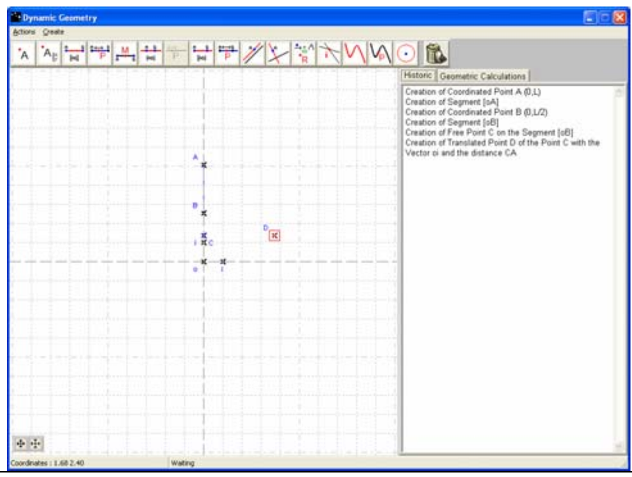


Create the *translated point D* a distance *L-d(o,C)* in the direction of the vector *oi*
The procedure for creating such points is:

1. Choose the menu *Create -> Point -> algebraic translated point*
2. Click on the point of reference *C*
3. Choose the origin of the vector *o* then the 2nd point *i*
4. Enter the distance *L-d(o,C)* in the dialog box.
   (Alternatively, you can enter *d(C,A)*)





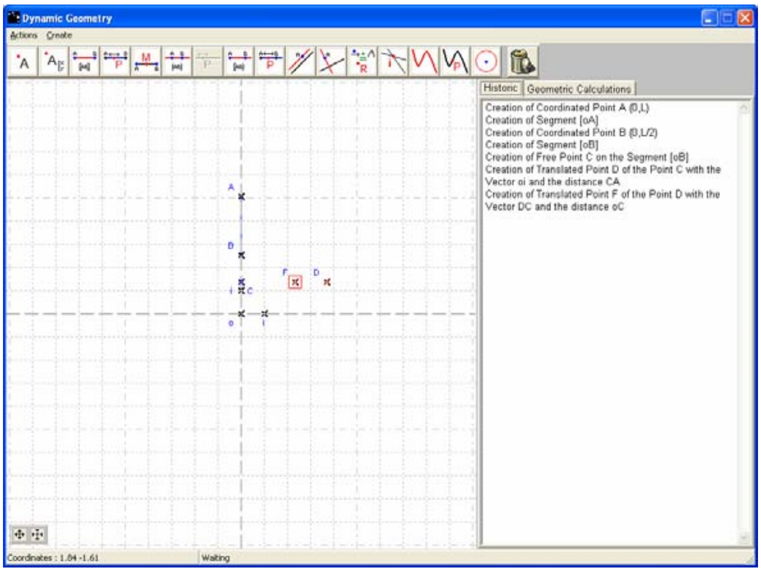


Alternatively you can use the menu item *Translated point* and follow the hints (click successively on C, o, I, C, A

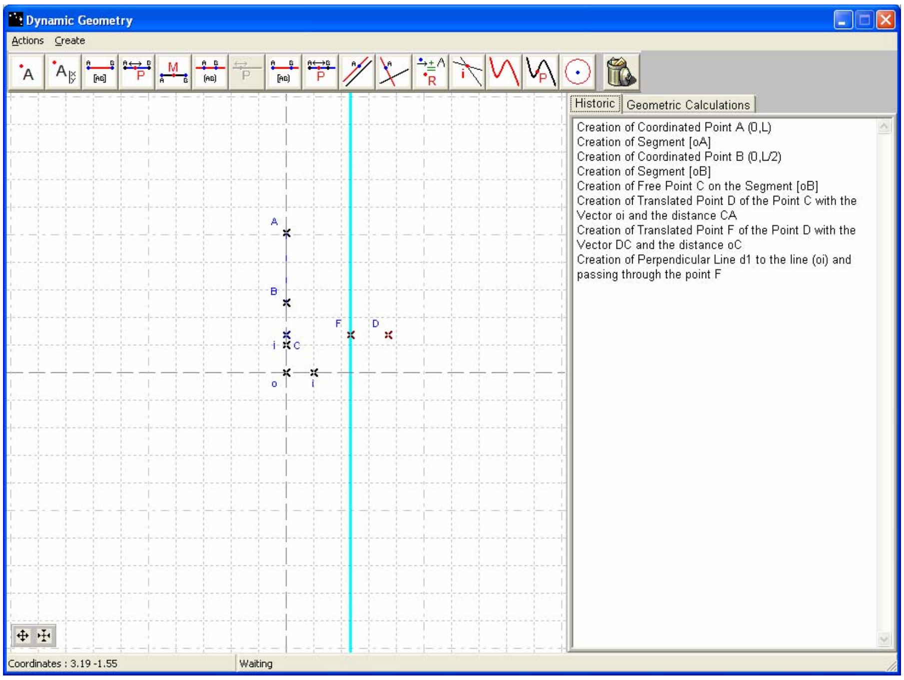
Create the *translated point* F a distance oC on the vector DC
The procedure for creating such points is:

1. Click on the point of reference *D*
2. Choose the first point on the vector *D* then the 2nd point on the vector *C*
3. Choose the 1st point to define the distance *o* and then the 2nd point to distinguish the end point for the distance *C*

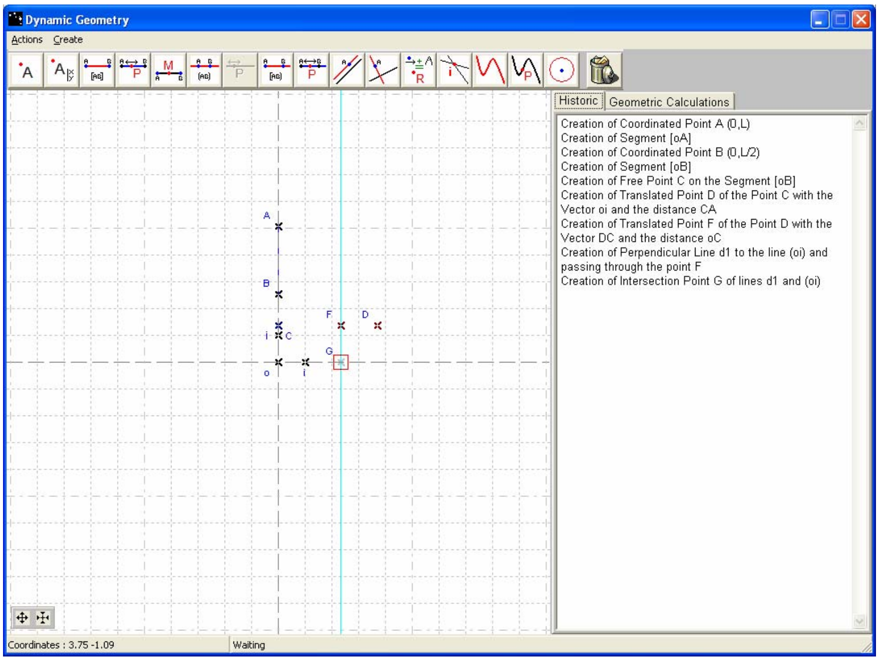
Alternatively you can use the menu item *algebraic translated point* to create the point F translated from C in the direction of vector oi, a distance L-2 d(o,C)



Create a *line perpendicular* to the x axis passing through the point F by the menu Create -> Line-> Perpendicular Line (alternatively you can use the corresponding Button in the tool bar). Click on point F, then on the x axis.

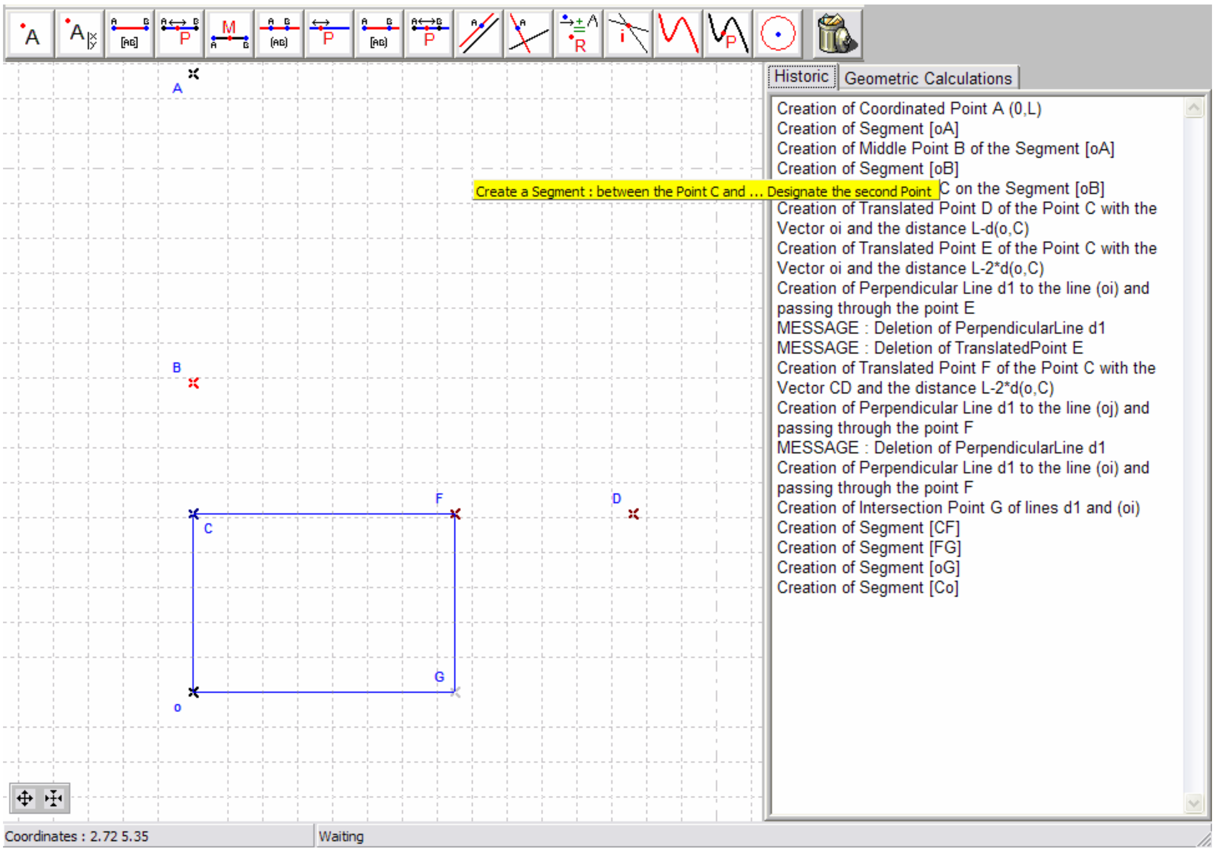


Create the point G the *intersection* of the line above and [oi] by the menu Create ->Point -> Intersection Point (alternatively you can use the corresponding Button in the tool bar). Click on the line, then on the x axis :



Zoom in (second button, bottom left) and drag the axis.

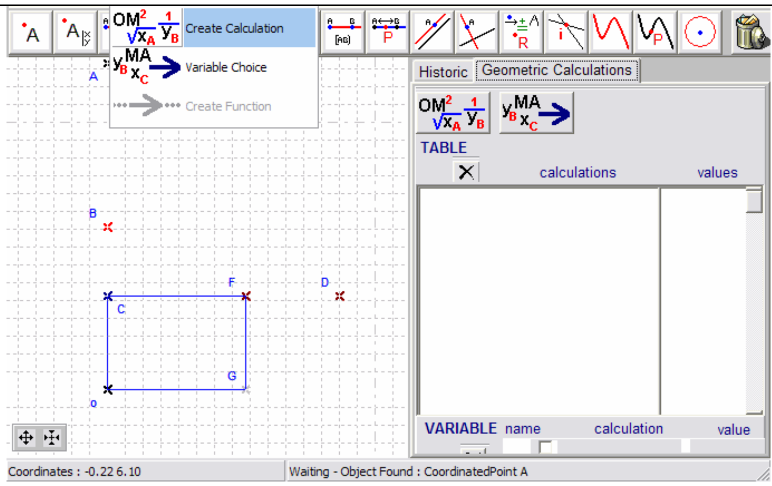
Hide the line, the axis, segments [oA] and [oB], points i, j (action ->Hide/unhide an object).
Create the segments [oC] [CF] [FG] [Go]
Then move point C to observe how the rectangle varies.

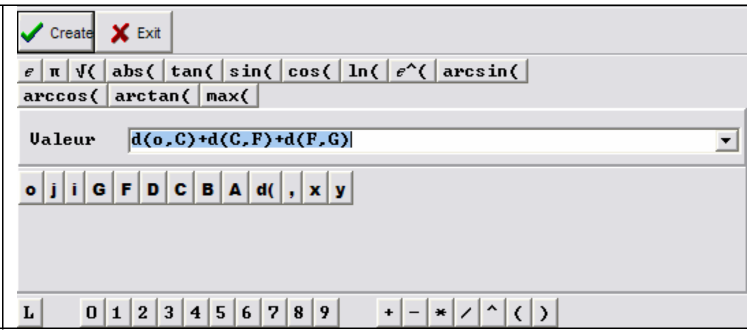


Save your figure by the file->save menu of the Casyopée symbolic window (you can use the Go to Casyopée button, if this window is not visible.

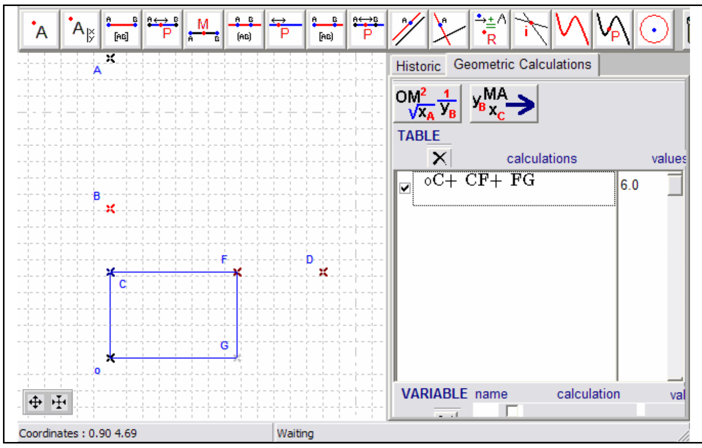
Creating Calculations

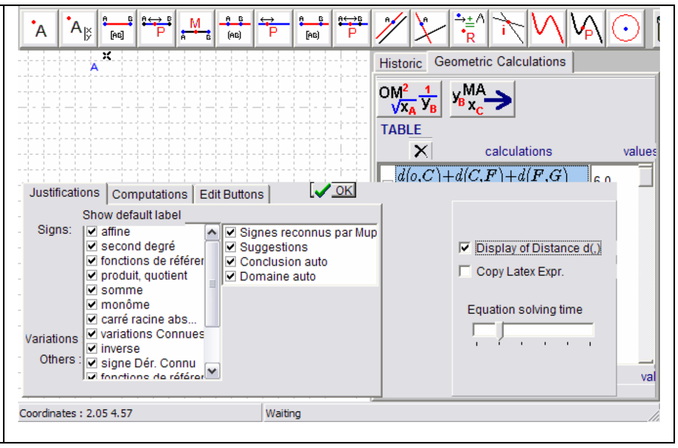
On the left panel, switch to the Geometric Calculation page. Activate the menu *calculations -> Create calculations* or the corresponding button on the Geometric Calculation page. The form to enter the geometric calculation window is open. Enter the sum d(o,C) + d(C,F)+ d(F,G) (or more simply oC+ CF +FG) which represents the sum of the lengths of the sides of the enclosure (refer to Figure 1 above).



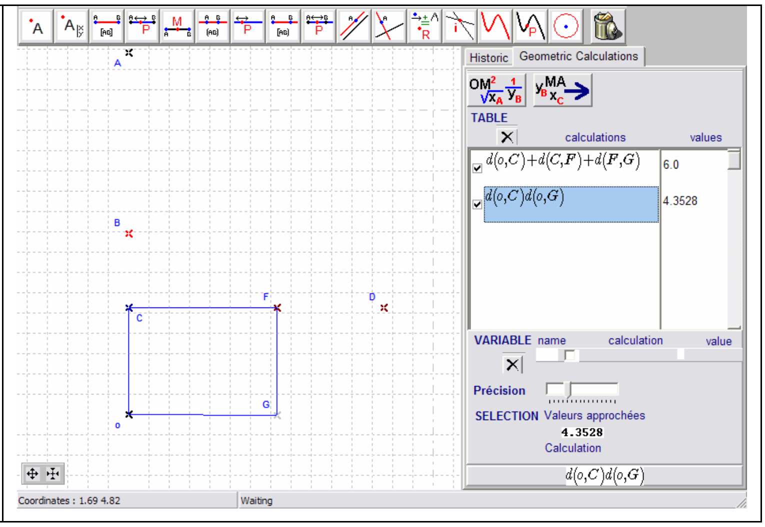


After clicking create and exit, the calculation is displayed in the calculation list on the left. Check the ticks box if you want to have the current approximate value. Note that this value is equal to the current value of the parameter L and does not change when you move C.
If you prefer to have the calculation in the form d(o,C) + d(C,F)+ d(F,G), tick the appropriate box in the Options form accessible by the Option->details menu of the symbolic window: after activating the option, delete (button above the list) and recreate the calculation.





Now create the calculation "oC x oG", which represents the area of the rectangle.
As this rectangle represents the enclosed field, we should be able to find the position of C such that this area is the maximum. Check that its approximate value evolves/moves when you move the point C. It must be zero at the two ends of the segment [oB] and pass through a maximum value when the point C is at the middle of the segment [oB].



Creating a Geometric Function

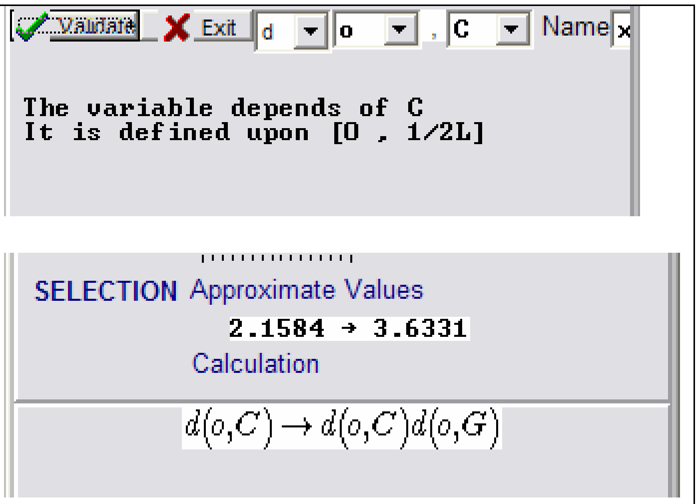
We will now deal with the problem of the maximum area in a symbolic way. In a symbolic system we need to define a function whose variable depends on the point C and whose value

is the area of the enclosed field. The Geometric Calculation in Casyopée will help us to do that.
Activate the menu *calculations ->choose variable* or the corresponding button on the Geometric Calculation page. Four lists appear. The first enables us to choose the type of measurement (distance, x-coordinate, y-coordinate). The next two enable us to choose the points concerned. The last one enables us to choose the identifier of the variable (x, *t*, *n*...)

There are several variables, which we can use for this problem. Let us choose the distance oC (one could also choose BC, or y_C) and the identifier *t*.

Casyopée declares that the variable is appropriate and returns the definition set ([0; L/2]). Validate this.

Note that the bottom of the window displays the co-dependency between the variable and the selected calculation in approximate and symbolic forms.
By moving C, you can now observe this co variation.

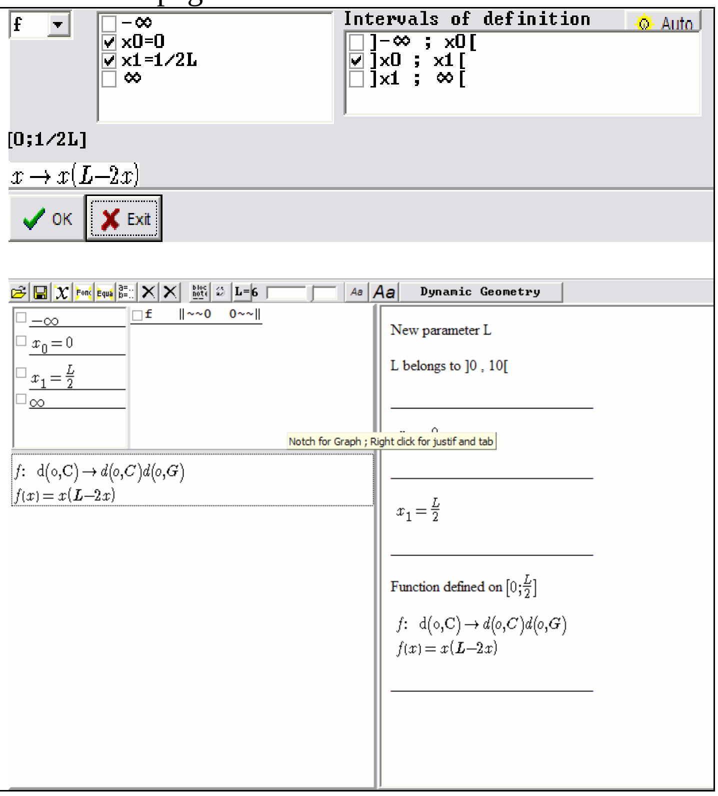


Now we will determine whether this co-variation is a functional dependency and, if it is, the algebraic form of the function. Activate the menu *calculations ->create function* or the corresponding button on the Geometric Calculation page.

Because, in this example, there is a functional dependency, an entry form opens. The algebraic form and the domain of the variable are displayed. You can choose the name of the function in the top left list.

Because the function is defined upon a subset of this domain, but not necessarily on the whole domain, you can amend this domain. For this example, it is not necessary. Then just activate OK.

Switch to the symbolic window of Casyopée to observe how the function has been created.



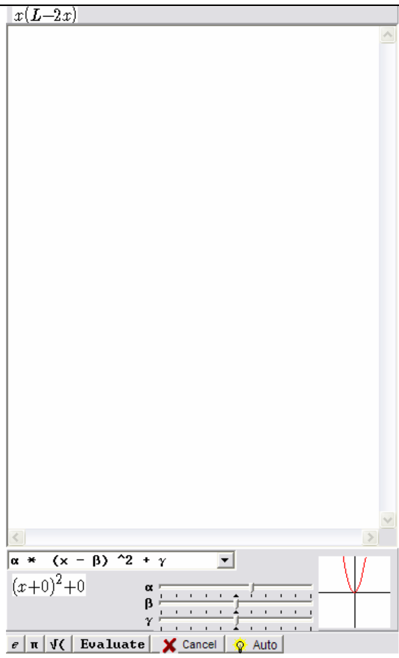
This is an outline of a symbolic study.
Graph the function by ticking the box near f in the list of function.
Select *f* in this list and activate *Justify-> variations->Fonctions de référence.*

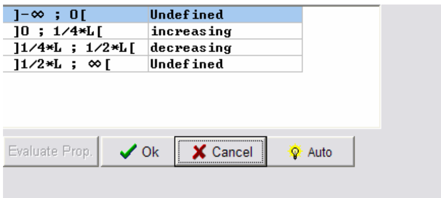
Choose the second item of the list at the bottom of the window. It corresponds to a quadratic function under the “completed square” form. You can move the sliders and try to adjust the values of the parameters to get a graph approaching the graph of *f*.

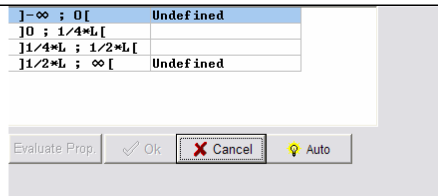
You can also evaluate and Casyopée will give you the “completed square” form, and help you to determine the variations of the function.

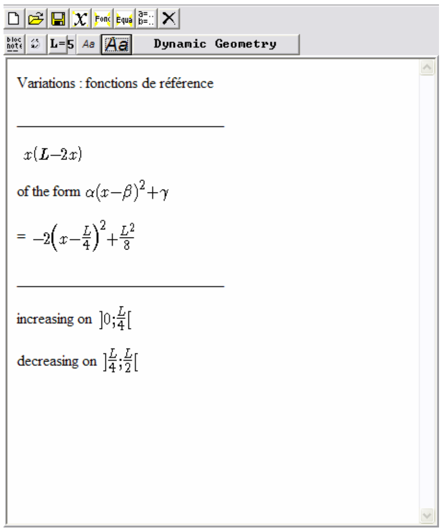
A summary is given in the notepad. It can be a basis for writing a formal proof.

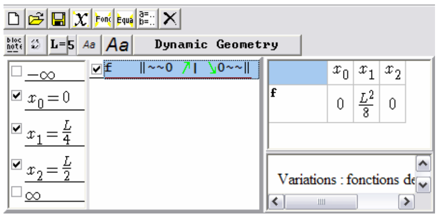
The list of functions gives a visual summary, and the symbolic values table gives the maximum area: *L²/8*.









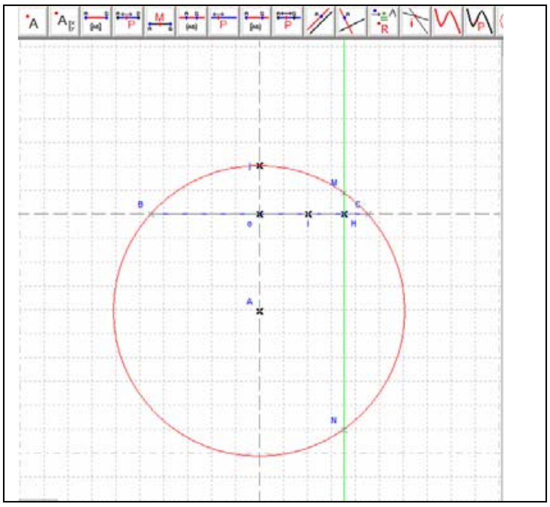


You can now switch back to the Dynamic Geometry and build the optimal fence by placing C at the middle of [oB] !

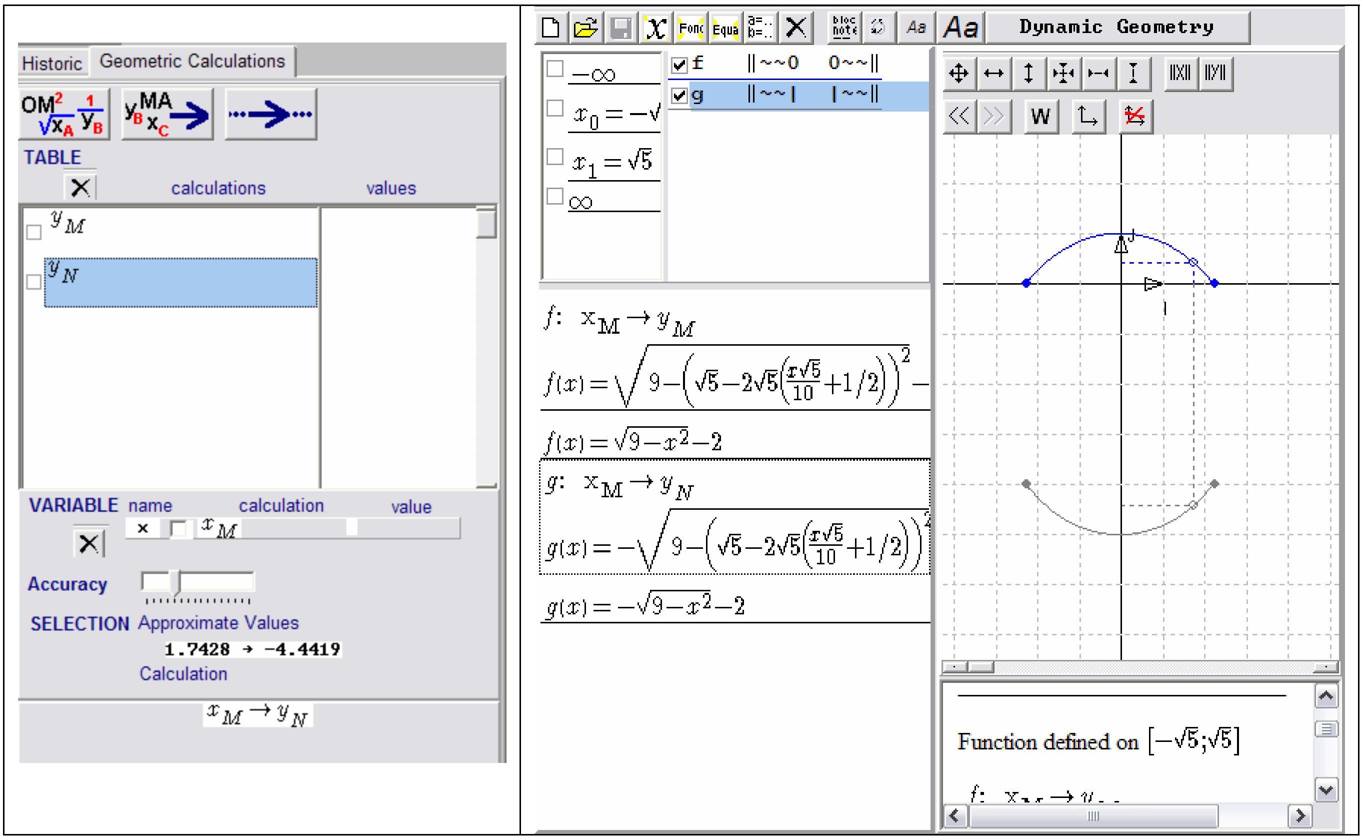
Casyopée Dynamic geometry extension (2): Locus and equations

Equations of Circles

Does a point moving on a circle define a function?
Create the point A (-2,0) then the circle centered in A passing by j.
Create the intersection points B and C between this circle and the x-axis.
Create the segment [AB] then a free point H on this segment.
Create the line passing by H and parallel to the y-axis.
Create the intersection points B and C between this line and the circle.

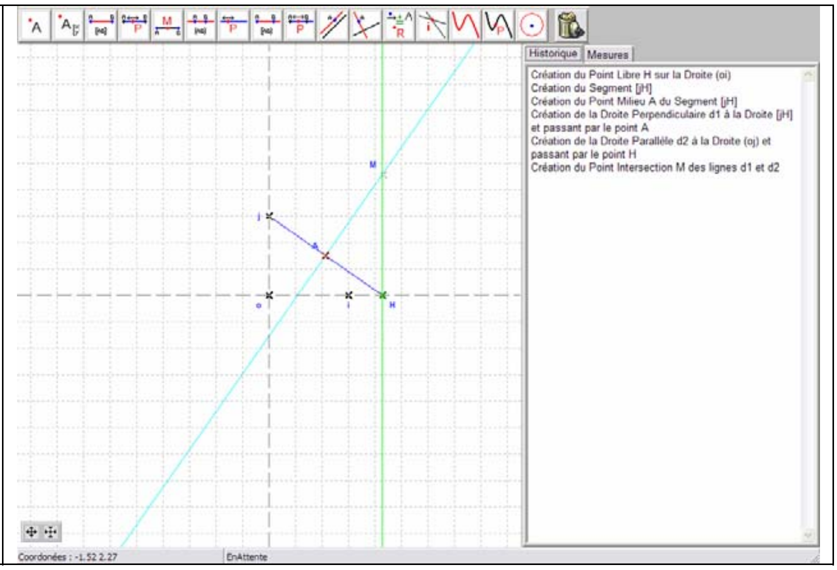


Choose the variable xM.
Create the calculations yM and yN.
Then create the corresponding functions.
The expressions for the functions are bulky. Use calculate->expand to get a simpler form.
Graph the functions and observe that when you move H, two points are moving on the graph.

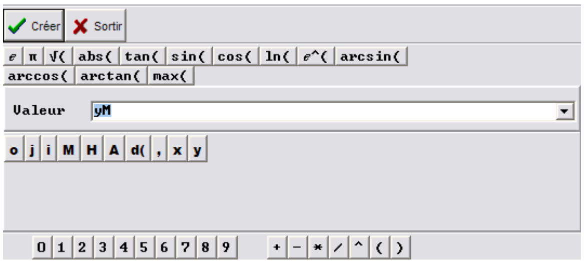


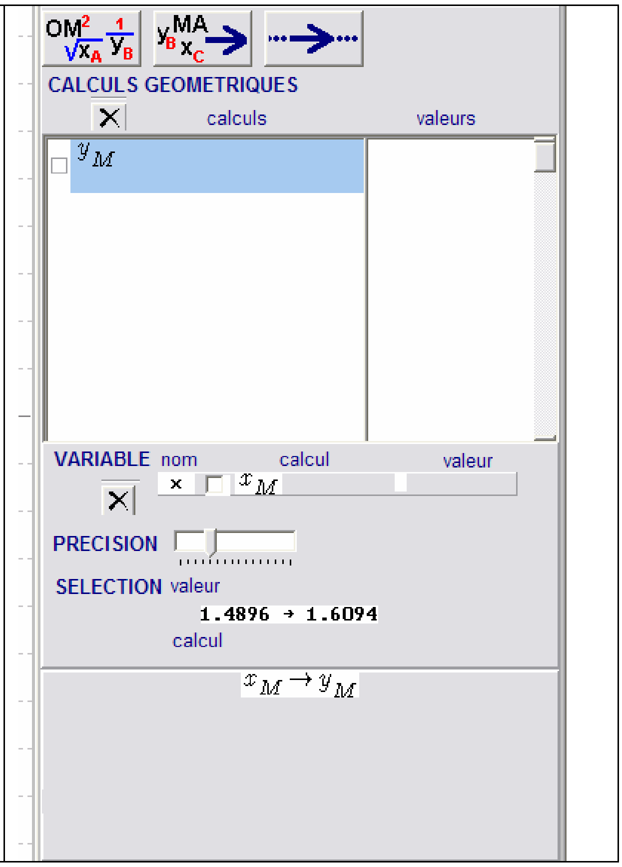
Equations of Parabolas

Create a free point on the line (oi). Name it H.
Create the segment (ij) an its middle.
Create the perpendicular passing by this middle to the segment.
Create the parallel passing by H to (oj).
Create the intersection between these two lines.
Name it M.

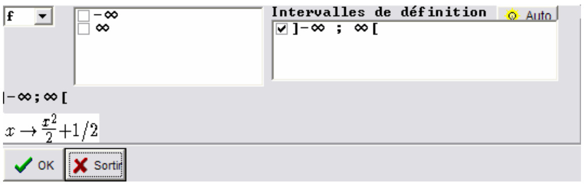


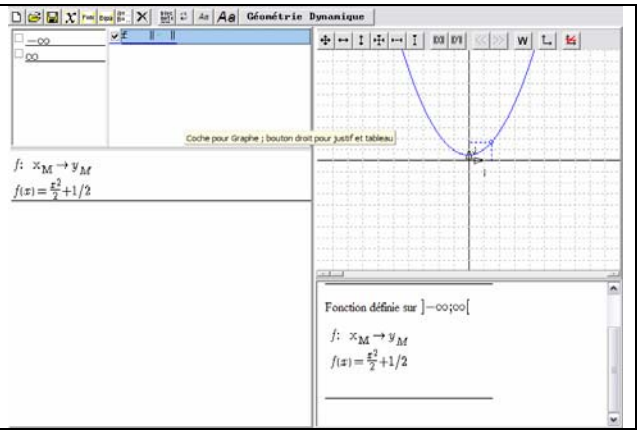
Create the calculation yM and choose the variable xM.
Observe the dependency between these coordinates of M.



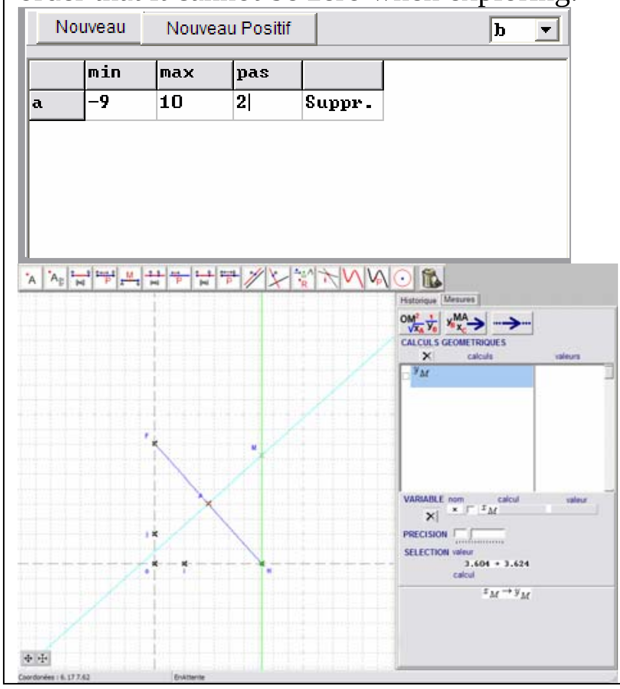


Create the function and graph.

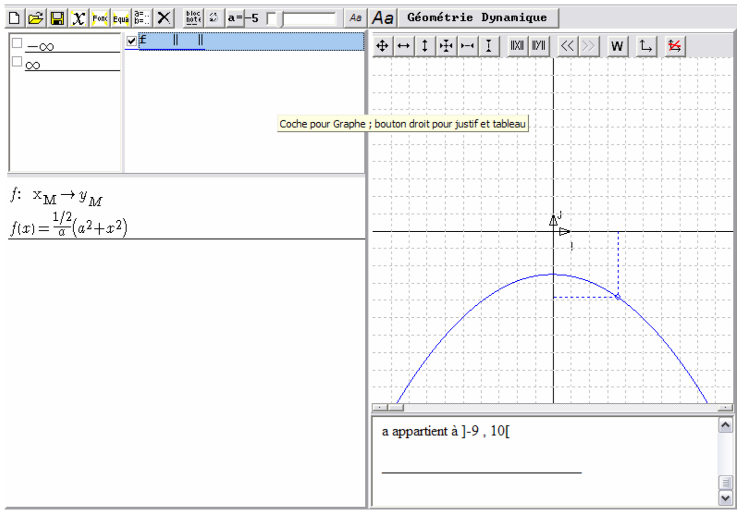




Try again by replacing point j by a coordinate point F (0, a), a being a Casyopée parameter. Be careful to define the variations of a, in order that it cannot be zero when exploring.

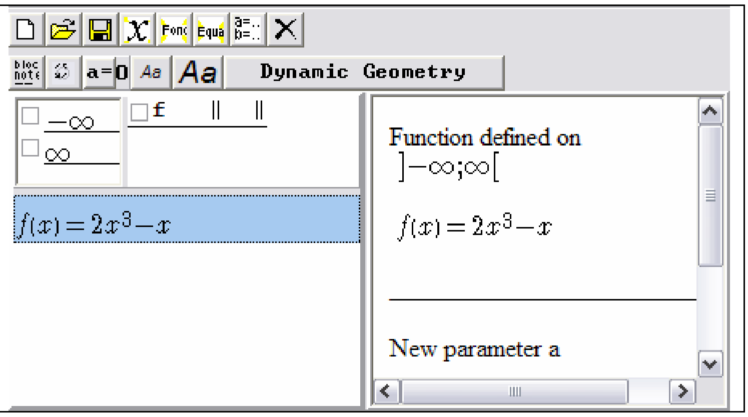


Explore the behaviour of the curve by animating a.

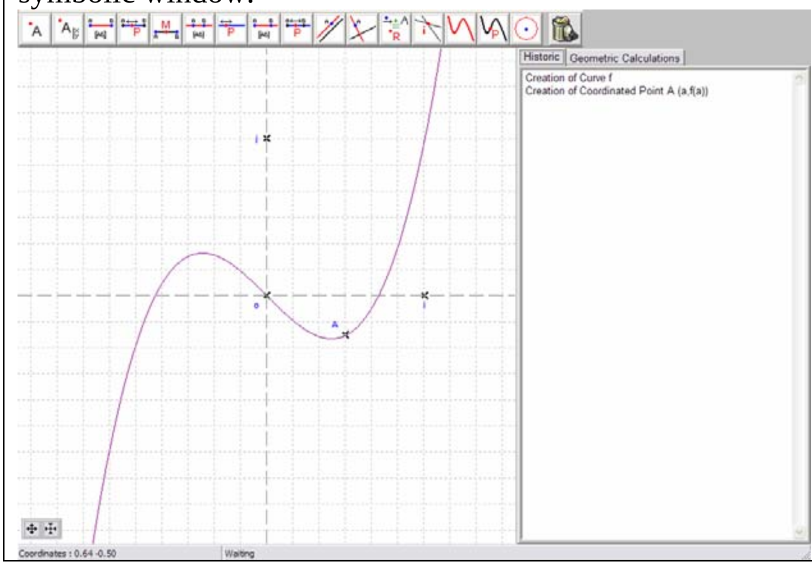


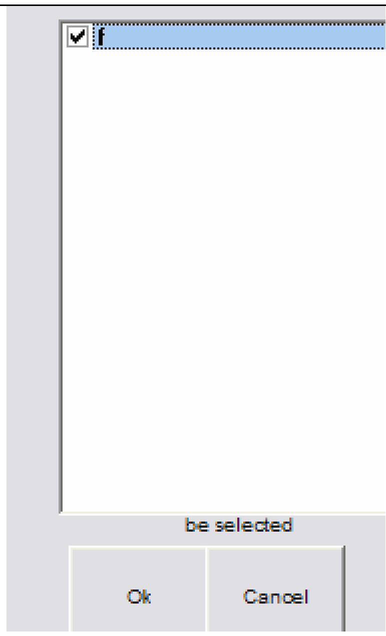
Casyopée Dynamic geometry extension (3): Handling a function as a geometrical object in the Dynamic Geometry extension

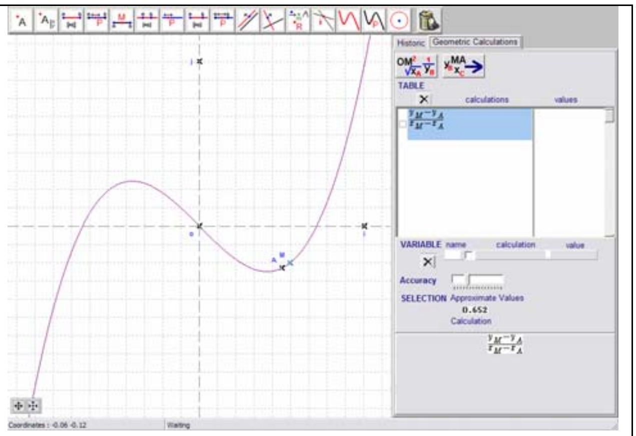
The goal is to study the differential quotient for a point on the graph of a cubic function.
First create the function x-> 2 x^3-x and a parameter a in the symbolic window.



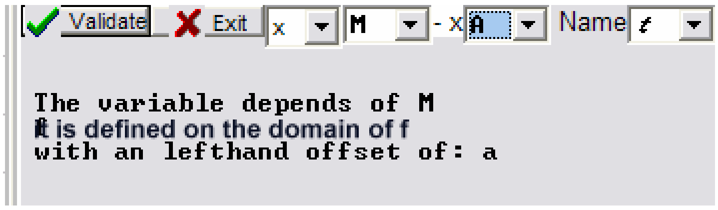
Then switch to the Dynamic Geometry window and activate the menu *create->curve* or the corresponding button. Tick the box near f, and click OK. The curve representing f is displayed.
Then create the coordinate point A (a, f(a)). Note that A moves on the curve when you animate the parameter a in the symbolic window.



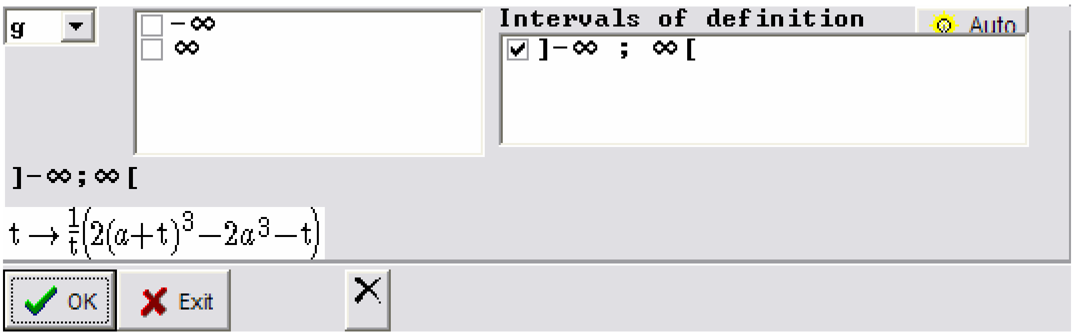


Create M, a free point on the curve by activating the menu *Create->Point, Free point on a curve* or the corresponding button. Note that M moves on the curve when you drag it with the mouse. Create the calculation (yM-yA)/(xM-xA) corresponding to the differential quotient relative to A. Observe the values of the quotient when M is near A for various values of a.	

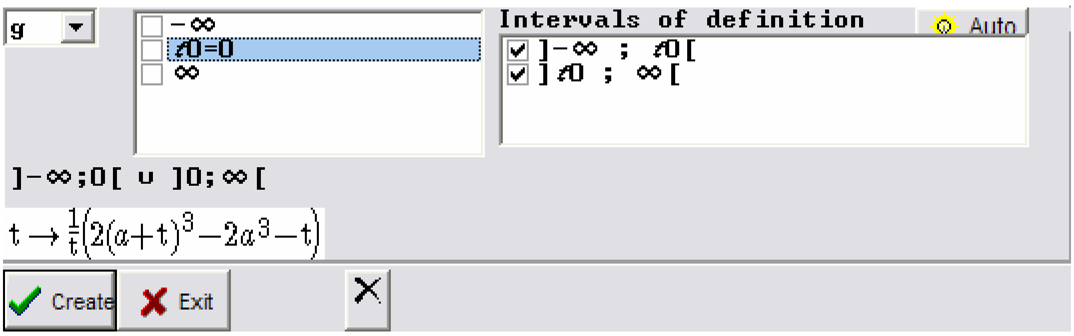
Then choose the variable xM-xA. Name it *t* in order to avoid confusion with x (the variable of the function f.



Then create the function. Note that the function is not defined for *t*=0. Thus if you validate, Casyopée will issue an error message.



To avoid this, create the value t0=0 by activating the menu *create*. Then untick the box near t0.



Now you can validate. Observe that the created function is not defined for t0=0, but it has a limit 6a²-1, which corresponds to the derivative of f(a) in a.